Higher order control Lyapunov functions and new sufficient stabilizability conditions for nonlinear control systems

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We consider an optimal control problem where the state has to approach asymptotically a closed target, while paying an integral cost with a nonnegative Lagrangian. We generalize the dissipative relation that usually defines a Control Lyapunov Function (CLF) by introducing a weaker differential inequality, which involves both the Lagrangian and higher order dynamics' directions expressed in form of iterated Lie brackets up to a certain degree k, possibly greater than 1. In particular, we show that the existence of a solution U of the resulting extended relation allows us to construct explicitly a Lie-bracket-based feedback which sample stabilizes the system to the target and, at the same time, regulated the cost, namely, provides a bound for the cost, given by a U-dependent function.

These results, on the one hand, extend previous results on global asymptotic controllability and stabilizability by considering both higher order conditions and a regulated cost. On the other hand, they can also be regarded as a generalization to exit time problems, possibly with a nonnegative vanishing lagrangian, of some classical and more recent small time local controllability conditions with the associated regularity (e.g., Hölder continuity) results for the minimum time function. Examples are provided where a smooth CLF fails to exist, while the distance from the target happens to be a smooth degree-2 CLF.

This talk is based on a joint work with Giovanni Fusco and Franco Rampazzo and is part of an ongoing, wider investigation of global asymptotic controllability and stabilizability in an optimal control perspective (which also involves Anna Chiara Lai, Univ. Roma La Sapienza).