CONTROLLABILITY AND INITIAL DATA IDENTIFICATION FOR CONSERVATION LAWS WITH SPACE DISCONTINUOUS FLUX

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In this presentation we will focus on controllability properties for the scalar conservation law

$$\partial_t u + \partial_x f(u,x) = 0 \tag{1}$$

where f is a discontinuous function in the space variable:

$$f(u,x) = \begin{cases} f_l(u), & x < 0, \\ f_r(u), & x > 0. \end{cases}; \qquad f_l, f_r \quad \text{strictly convex, smooth maps}$$

Conservation laws with discontinuous flux have numerous applications; two well known examples are traffic flow with heterogeneus road conditions and two phase flow in porous media.

Motivated by the study of controllability and regularity properties, in the first part of the talk we shall discuss some adapted Oleinik estimates.

The second part of the talk will be devoted to the problem of initial data identification. Namely, for $\omega \in L^\infty(\mathbb{R})$ and $T > 0$, our goal will be to characterize the set of initial data that yield the fixed profile ω after time $T > 0$. More precisely, given the set

$$\mathcal{I}_T^{AB}\omega = \{u_0 \in \mathbb{L}^\infty \ : \ \mathcal{S}_T u_0 = \omega\},$$

where $\mathcal{S}_t u_0$ is the unique solution with initial datum equal to u_0, we will study its geometrical and topological properties.

This is a joint research with Fabio Ancona (Università di Padova).

References

[1] F. Ancona, M. T. Chiri, *Attainable profiles for conservation laws with flux function spatially discontinuous at a single point*, ESAIM: COCV, Vol. 26, (2020), pp 1–33.

[2] F. Ancona, M. T. Chiri, L. Talamini, *Correction to: Attainable profiles for conservation laws with flux function spatially discontinuous at a single point*, in preparation.

[3] F. Ancona, L. Talamini, *On the attainable set for conservation laws with space discontinuous flux and time dependent connections*, in preparation.

[4] F. Ancona, L. Talamini, *Initial data identification for conservation laws with space discontinuous flux*, in preparation.