Parking with state constraints

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Abstract

Let us consider a non-linear, control-affine, system $\dot{x} = \sum_{i=1}^{m} g_i(x)u^i$ whose state x is constrained in the closure $\bar{\Omega}$ of an open set $\Omega \subset \mathbf{R}^n$. For many goals the following property **(P)** is crucial:

(P) *Every system trajectory* $x : [0,T] \to \mathbf{R}^n$, $x(0) \in \bar{\Omega}$, *possibly violating the constraint, can be approximated by a new system trajectory* $\hat{x}(\cdot)$ *that satisfies the constraint and whose distance from* $x(\cdot)$ *is bounded by a quantity proportional to*
$d := \sup\{dist(\Omega, x(t)),\ t \in [0,T]\}$.

While in general this condition is not verified, it turns out to be crucial for many goals. For instance, **(P)** implies the continuity of the value function, which in turn is sufficient for the uniqueness of the solution of the associated Hamilton-Jacobi-Bellman equation, an essential condition e.g. for numerical schemes. The standard hypothesis guaranteeing that property **(P)** is verified is the so called *inward pointing condition* (IPC), prescribing, at each point of the boundary $\partial\Omega$, a choice of the control u such that the dynamics points strictly inside Ω (the mere tangency beeing not enough).

I will describe a new condition $(\mathrm{IPC})_{Lie}$, weaker than (IPC), which consists in a sort of inward pointing condition involving the Lie brackets $[g_i, g_j]$, $i, j = 1, \ldots, m$, together with a non-positive-curvature assumption. Under hypothesis $(\mathrm{IPC})_{Lie}$, the implementation of a suitable rotating control strategy allows for a novel construction of a constrained trajectory $\tilde{x}(\cdot)$ whose distance from $x(\cdot)$ is now bounded by a quantity proportional to $\sqrt{d}$. The latter property is slightly weaker

than **(P)** but it is still sufficient most of the many goals achieved by assuming **(P)**.

(The mildly challenging title refers to the fact that, on the one hand, the so called *parking problem* is a toy example where the direction of the Lie bracket is crucial to have full controllability, and, on the other hand, the presence of state constraints makes the usual computations of geometric control useless.)

References

[1] G.Colombo, N. Khalil, and F.Rampazzo: *State constraints, higher order inward pointing conditions, and neighboring feasible trajectories*, To appear in SIAM Journal on Control and Optimization.