Hölder and Markov capacities in $\mathbb{C}^N$

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Abstract. Let V_E be the pluricomplex Green function associated to a compact subset E of $\mathbb{C}^N$. The well known Hölder Continuity Property of E means that there exist constants $A > 0, \gamma \in (0,1]$ such that

$$|V_E(w) - V_E(z)| \leq A\,|w - z|^\gamma.$$

It turns out that this condition is equivalent to a Vladimir Markov type inequality, i.e.

$$\|D^\alpha P\|_E \leq M^{|\alpha|}\,(\deg P)^{m|\alpha|}\,|\alpha|!^{1-m}\,\|P\|_E,$$

where $m, M > 0$ are independent of the polynomial P of N variables and $\|\cdot\|_E$ is the supremum norm on E. In this context we give a definition of Markov and Hölder capacities that are strictly related to the best constants in Markov and Hölder inequalities. We will present some links between these capacities and the L-capacity of E given by

$$C(E) = \liminf_{||z||_2 \longrightarrow \infty} \frac{||z||_2}{\exp V_E^*(z)}.$$