An efficient algorithm to model dynamical system's separatrix manifolds

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Mathematical modelling is being applied in major disciplines, such as biology, medicine and social sciences and the aim of such models lies in the prediction of the temporal evolution of the considered quantities. The possible steady states of the system are completely determined by its parameters. However, it is possible that more steady states can originate from the same parameter set. In that case the outcome of such model depends on the initial condition. Therefore, it is important to assess the domain of attraction for each possible attractor. Here we focus on systems of two and three ordinary differential equations aimed at finding the basins of attraction of three different equilibria. For this purpose we have implemented several MATLAB functions for the approximation of the points lying on the curves or on the surfaces determining the basins of attraction, obtained by a bisection algorithm, and for the graphical representation of such curves and surfaces. In particular our algorithm is general and robust enough so as to accommodate the three equilibria case as well as the particular case in which two of these equilibria coincide, improving the results obtained in [1, 2, 3]. Moreover, we approximate the curves and the surfaces with the implicit Partition of Unity method, using as local approximants the compactly supported Wendland's functions, [4, 5].

References

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