A Nyström method for integral equations with Mellin type kernels

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Abstract

We consider integral equations of the following type

$$f(y) + \int_0^1 k(x,y) f(x) dx + \int_0^1 h(x,y) f(x) dx = g(y), \quad y \in (0,1], \tag{1}$$

where $k(x,y) = \pm\frac{1}{x}\bar{k}\left(\frac{y}{x}\right)$ is a Mellin type kernel with $\bar{k} : [0,+\infty) \to [0,+\infty)$ a given function satisfying suitable assumptions, $h(x,y)$ and g are given smooth functions and f is the unknown.

Many problems in mathematics, physics and engineering give rise to such kind of integral equations. For example they occur when boundary integral methods are used on domains with corners.
They are characterized by the presence of a non-compact Mellin convolution operator with kernel having a fixed-point singularity at $x = y = 0$. Then the standard stability proofs for numerical methods do not apply and a modification of the classical methods in a neighbourhood of the endpoint $x = 0$ is needed.

We propose to approximate the solutions of (1) by a "modified" Nyström method using the Gauss-Legendre quadrature rule. The modification of the classical method that we introduce is new and is based on a suitable approximation of the integral transform

$$(Kf)(y) = \int_0^1 k(x,y) f(x) dx$$

in points very close to 0.

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The stability and the convergence are proved in L^2 spaces and error estimates in L^2 norm are given. Finally, numerical tests showing the effectiveness of the method are presented.