Numerical methods for bivariate Fredholm Integral Equations

D. Occorsio [a], M.G. Russo [a]

[a] *Department of Mathematics and Computer Science, University of Basilicata, v.le dell'Ateneo Lucano 10, 85100 Potenza, Italy*

Abstract

We investigate the numerical solution of two-dimensional Fredholm integral equations defined on the set S,

$$f(x,y) - \mu \int_S k(x,y,s,t) f(s,t) \mathbf{w}(s,t)\, ds\, dt = g(x,y), \quad (x,y) \in S,$$

where S can be a bounded or unbounded domain of the plane, $\mathbf{w}(x,y) := w_1(x)w_2(y)$ and w_1, w_2 are suitable weight functions, μ is a real number.

k and g are given functions defined on S^2 and S respectively, which are sufficiently smooth on the open sets but can have (algebraic) singularities on the finite boundaries and an exponential growth at $\pm\infty$ at most w.r.t. each variable if the domain is unbounded. f is the unknown function.

We introduce some Nyström methods based on cubature formulas obtained as tensor products of two Gaussian quadrature formulas w.r.t. the weights w_1, w_2. In the case of unbounded domains we need to "truncate" the quadrature rules. The convergence, stability and well conditioning of the methods are proved in suitable weighted spaces of continuous functions.

Some numerical examples illustrate the efficiency of the methods.

Email addresses: donatella.occorsio@unibas.it (D. Occorsio), mariagrazia.russo@unibas.it (M.G. Russo).