Pricing American multi-asset options using radial basis function partition of unity method

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American-style options are the most traded options on exchanges. The price of an option can be found by solving famous Black-Scholes partial differential equation, which becomes multi-dimensional in case the option is written on several assets. In addition, it turns out that pricing of American-style options is a problem involving a free boundary. We consider two different approaches of handling the free boundary. One is the penalty method [3] that allows to convert the original free boundary problem into a fixed domain problem, but introduces an error which depends on the size of the chosen penalty. This method has been successfully used together with radial basis function (RBF) methods [5]. The second one is the operator splitting (OS) method [2], which in fact solves the linear complementarity problem. This approach has been shown to be an efficient way of pricing American options, and has mostly been used together with finite differences (FD), but not with RBFs. We implement the OS method with RBF methods in order to price American multi-asset options and compare the results with their penalty counterparts.

The global radial basis function (RBF) method is known by its high order polynomial or for smooth problems spectral convergence and by its ability to work on non-regular meshes [4]. A drawback of the global method is that it leads to a dense ill-conditioned linear system, and therefore it becomes problematic to use the global RBF method for high-dimensional problems, where the number of degrees of freedom grows exponentially. In order to overcome this issue we employ a partition of unity (PU) technique [1], which allows to significantly sparsify the system and, hence, reduce the computational effort, while maintaining high accuracy.

Our idea is that a combination of the OS method and highly accurate RBF-PU method will result in a good tool for pricing American multi-asset options.

References

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