The RBF-QR method and its applications

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In these two lectures, I will give a background to the development of stable methods including the flat limit behavoiur of radial basis function (RBF) approximations [3, 8, 12, 11] and an overview of stable methods such as the Contour-Padé approach [7], the RBF-GA method, and RBF-QR methods with more detail for the latter, including the RBF-QR method on the sphere [6] that was the first RBF-QR method, the RBF-QR method in Cartesian space [5, 9], and the different formulations introduced in [4, 1, 10, 2]. Relevant in this context is also node placement, and different ways in which this affects approximation performance.

Furthermore, I will give arguments for why stable methods are necessary, in particular when localized RBF methods are concerned. Exemples with stencil-based methods (RBF-FD) and partition of unity-based methods (RBF-PUM) will be given. Theoretical and numerical results for different scaling strategies and applications will be used to illustrate the point. Finally, simulation result for the solution of partial differential equations in application areas such as atmospheric science and finance will be shown.

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