

# Approximation of pluricomplex Green functions based on Monte Carlo integration

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## Abstract

We approximate the pluricomplex Green functions for pluriregular compact sets that satisfy the Bernstein-Markov property using Monte Carlo methods. They provide a suitable approximation of the Gram matrices of bases for the spaces  $\mathcal{P}_k(\mathbb{C}^N) \cap L^2(\mu)$ , where  $\mu$  is the measure from the Bernstein-Markov property, and  $\mathcal{P}_k(\mathbb{C}^N)$  denotes the space of polynomials of degree at most  $k$ . The approximative Gram matrices lead to approximation of orthonormal bases in  $\mathcal{P}_k(\mathbb{C}^N) \cap L^2(\mu)$  and hence to the Bergman functions. The latter can be used to approximate the pluricomplex Green functions in view of the Bloom-Shiffman formula. Such approximation can be slow in some cases as we show on the example of the  $N$ -asterisk in the complex plane in which case the Bergman functions can be calculated explicitly and the Monte Carlo integration is not required. Using the Monte Carlo approach we provide some examples and examine errors for sets including a square, a hexagon and a pentagon in  $\mathbb{R}^2 \subset \mathbb{C}^2$ .

This is a joint research project with Maciej Klimek, Department of Mathematics, Uppsala University (Sweden).

## References

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