A periodic map for linear barycentric rational trigonometric interpolation

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Let \( \phi \) be the set of equidistant nodes in \([0, 2\pi)\),

\[
\phi_k = \frac{2\pi k}{n}, \quad k = 0, \ldots, n - 1.
\]

(1)

For an arbitrary \( 2\pi \)-periodic function \( f(\phi) \), the formula for the corresponding trigonometric barycentric interpolant between the \( \phi_k \)'s is

\[
T[f](\phi) = \frac{\sum_{k=0}^{n-1} (-1)^k \cst(\frac{\phi - \phi_k}{2}) f_k}{\sum_{i=0}^{n-1} (-1)^i \cst(\frac{\phi - \phi_i}{2})},
\]

(2)

where \( \cst(\cdot) := \text{ctg}(\cdot) \) if the number of nodes \( n \) is even, and \( \cst(\cdot) := \csc(\cdot) \) if is odd.

Baltensperger [1] has shown that the corresponding barycentric rational trigonometric interpolant (2) introduced in [2] converges exponentially toward \( f \) when the nodes are the images of the \( \phi_k \)'s from (1) under a periodic conformal map.

In the present work, we present a simple periodic conformal map which accumulates nodes in the neighborhood of an arbitrarily located front, as well as its extension to several fronts. Despite its simplicity, this map allows for a very accurate approximation of many functions with steep gradients.

References
