A periodic map for linear barycentric rational trigonometric interpolation

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Let $\boldsymbol{\phi}$ be the set of equidistant nodes in $[0, 2\pi)$,

$$\phi_k = \frac{2\pi k}{n}, \qquad k = 0, \ldots, n-1. \tag{1}$$

For an arbitrary 2π–periodic function $f(\phi)$, the formula for the corresponding trigonometric barycentric interpolant between the ϕ_k's is

$$T[f](\phi) = \frac{\displaystyle\sum_{k=0}^{n-1} (-1)^k \ \mathrm{cst}(\frac{\phi - \phi_k}{2})\, f_k}{\displaystyle\sum_{i=0}^{n-1} (-1)^i \ \mathrm{cst}(\frac{\phi - \phi_i}{2})}, \tag{2}$$

where $\mathrm{cst}(\cdot) := \mathrm{ctg}(\cdot)$ if the number of nodes n is even, and $\mathrm{cst}(\cdot) := \csc(\cdot)$ if is odd.

Baltensperger [1] has shown that *the corresponding barycentric rational trigonometic interpolant* (2) *introduced in [2] converges exponentially toward f when the nodes are the images of the ϕ_k's from* (1) *under a periodic conformal map.*

In the present work, we present a simple periodic conformal map which accumulates nodes in the neighborhood of an arbitrarily located front, as well as its extension to several fronts. Despite its simplicity, this map allows for a very accurate approximation of many functions with steep gradients.

References

[1] R. Baltensperger, *Some results on linear rational trigonometric interpolation*, Comput. Math. Appl., 43 p.737-746 (2002),

[2] J.-P. Berrut, *Rational functions for guaranteed and experimentally well-conditioned global interpolation*, Comput. Math. Appl., 15(1) p.1-16 (1988),