A robust meshfree PDE solver for source-type flows in porous media

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Radial Basis Function (RBF)-based methods, taking advantage of being meshfree, are nowadays widely adopted tools for solving Partial Differential Equations (PDEs) via collocation schemes. Generally, the local approximants and consequently also the global ones may suffer from instability due to the ill-conditioning of the interpolation matrices. To avoid this drawback, which becomes even more evident when approximationg functions with singularities or discontinuities, we suggest a methodology consisting in building the differentiation matrices via the so-called Variably Scaled Kernels (VSKs). VSK were first introduced in [1]. In this framework, we propose an efficient and robust approach which turns out to be suitable in a *realistic* engineering problem where a steady state flow is assumed determined by a pulse-like extraction of water at a constant volumetric rate [2]. This leads to an elliptic PDE with a singular forcing term. In order to tackle such a type of flow configuration in porous formations, where the radial symmetry is lost, due to the heterogeneous soil properties, generally numerical schemes with fine grid level of discretizations near singularities are used. Conversely, we apply a *local* RBF scheme based on a partitioning of the domain. This approach is referred to as Partition of Unity (PU) [3]. A comparison with the performances of a classical grid-based scheme confirms the validity of the approach in solving such a real application problem.

References

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