Unsolved Problems in RBF Theory and Applications

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Since part of the DRWA audience is focusing on other fields like Mathematical Finance, Multivariate Polynomial Approximation, Imaging or Orthogonal Polynomials, the tutorial starts from the basics of Reproducing Kernel Hilbert Spaces and proceeds slowly towards the open problems. Among various smaller items, the following problem areas shall be in the focus:

- Metrics and geometry induced by general kernels,
- Analytic background of spaces with radial kernels, including pseudodifferential and extension/restriction operators,
- Scaling problems, a.k.a. choice of shape parameters,
- Boundary effects and superconvergence,
- The "Analytic Gap": why certain proofs fail for analytic RBFs.

Some of these are closely related, and the challenge is to uncover these relations.

The overall idea is to present a large number of interesting open problems for younger scientists to work on.