Meshless Methods for Solving PDEs

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Rather than giving a complete survey, the tutorial will present the author's way to bring some order into the chaos of the roughly 50 kinds of meshless or meshfree methods. Like in the first tutorial, the focus is on open problems.

The various so-called "forms" of PDE problems (strong, weak, locally weak etc., which are not forms of the same problem, but different problems) will first be unified by writing them as infinitely many constraints on an unknown solution. This abstract viewpoint includes both FEM-based and meshless methods, making comparisons on a common basis possible, and allowing a unified treatment of convergence and stability. Numerical stability can be deduced from analytical stability (well-posedness), and if stability holds, the error analysis and convergence rates can be reduced to how well trial spaces approximate the true solution. *Symmetric* methods of Rayleigh-Ritz type, realizing projectors in Hilbert space, are optimal, no matter if they are meshless (symmetric collocation) or non-meshless (classical FEM). For *unsymmetric* methods (Petrov-Galerkin or Kansa-type), oversampling will be shown to ensure stability.

Then the tutorial proceeds to techniques characteristic for meshless methods, namely being written *entirely in terms of nodes*, as the early survey of Belytschko et.al. stated. Still taking both meshless and non-meshless techniques into account, the focus turns to *localization* in its various forms, e.g. generalized finite difference methods like RBF-FD, Moving Least Squares, Partitions of Unity, and FEM-type triangulation-based trial spaces. These might work with or without fixed trial spaces in the background, and they sometimes take special care for a simplified numerical integration in the weak case, avoiding what is often called a "background mesh".

Along the way, various open problems will be pointed out. If time permits, excursions into nonlinear or time-dependent problems will be added.