Pythagorean-Hodograph curves in exponential polynomial spaces

Lucia Romani
University of Bologna

Abstract: Ordinary polynomial curve segments with the Pythagorean-Hodograph (PH) property have been extensively studied [2], and their construction has been satisfactorily extended also to spaces spanned by algebraic-trigonometric polynomials [1,4,5]. Although spaces spanned by algebraic-hyperbolic polynomials have close analogies with the ones spanned by algebraic-trigonometric polynomials (e.g, they are both special instances of Extended Chebyshev (EC) spaces [6]), their handling might require some additional caution. Indeed, even if the hyperbolic cosine and sine are just the opposite side of the exponential coin from the trigonometric cosine and sine, the normalized B-basis (also known as Chebyshevian Bernstein basis) of the underlying EC space is known to be affected by numerical instability when large exponential shape parameters are selected [6]. One of the goals of this talk is thus to suggest a stable formulation of the normalized B-basis of the two algebraic-hyperbolic spaces that are most frequently encountered, so that numerical instabilities are avoided. Moreover, for such spaces, we aim at proposing a novel evaluation algorithm that, compared with the dynamic evaluation procedure in [7], can be used with any choice of the shape parameter and, compared with the de Casteljau-like B-algorithm [3] (analogue of the de Casteljau algorithm for classical polynomial Bézier curves), is not a recursive corner-cutting anymore, but guarantees lower computational costs. Finally, after having introduced a stable and efficient procedure for evaluating algebraic-hyperbolic Bézier curve segments, we focus our attention on the constraints that the Bézier control polygon has to satisfy in order to enjoy the PH property. Mimicking the ordinary polynomial case, we provide an intuitive geometric characterization of algebraic-hyperbolic PH curves where the conditions are stated in terms of the geometry of their Bézier control polygons.

(This is a joint work with Alberto Viscardi, University of Bologna).

References

[1] I. Cattiaux-Huillard, L. Saini. Characterization and extensive study of cubic and quintic algebraic-trigonometric planar PH curves. Adv. Comput. Math. 46(2) (2020), article 28.

[2] R.T. Farouki. Pythagorean-Hodograph Curves: Algebra and Geometry Inseparable. Springer, Berlin (2008).

[3] E. Mainar, J.M. Peña. Optimal bases for a class of mixed spaces and their associated spline spaces. Computers and Mathematics with Applications 59 (2010) 1509-1523.

[4] L. Romani, F. Montagner. Algebraic-Trigonometric Pythagorean-Hodograph space curves. Adv. Comput. Math. 45(1) (2019) 75-98.

[5] L. Romani, L. Saini, G. Albrecht. Algebraic-Trigonometric Pythagorean-Hodograph curves and their use for Hermite interpolation. Adv. Comput. Math. 40(56) (2014) 977-1010.

[6]Á. Róth. Algorithm 992: An OpenGL- and C++-based function library for curve and surface modeling in a large class of extended Chebyshev spaces. ACM Trans. Math. Software 45 (2019), article 13.

[7] X. Yang, J. Hong. Dynamic evaluation of exponential polynomial curves and surfaces via basis transformation. SIAM Journal on Scientific Computing 41(5) (2019) A3401-A3420.