Multivariate polynomial approximation

Session at the *5th Dolomites Workshop on Constructive Approximation and Applications*

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List of contributions

Schedule

	Mon. 6th	Tue. 7th	Wed. 8th	Thu. 9th	Fri. 10th
10:30 – 12:30					
14:30 – 17:00				M. Baran S. Ma'u T. Beberok D. J. Kenne L. Białas-Cież	

Green functions via hypergeometric functions for a class of compact planar sets

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Abstract: We shall present how the idea of a radialization of complex Green function $V(E,z)$ gives a formula for the Green function for a class of compact planar sets E_m, $1 \leq m \leq 2$. E_1 is the unit circle, E_2 is the interval $[-1,1]$. We have $V(E_m,z) = V_m(z),\ z \in \mathbb{C} \setminus E_m$, where

$$V_m(z) = \Re\left((z-1)^{1/m}(m-1+z)^{1-1/m}F(1,1,1+\frac{1}{m},\frac{1-z}{m})\right),\ z \in \mathbb{C} \setminus [1-m,1),$$

$$E_m = \{z:\ V_m(z) = 0\}.$$

Here $F(a,b,c,z)$ denotes the classical Gauss hypergeometric function with parameters a,b,c.

We shall discuss properties of sets E_m, between them of logarithmic capacity. We obtain a formula, with classical digamma function as the main ingredient. Each E_m is HCP set with exponent $1/m$ and thus Markov's set with exponent m. The cusp $z = 1 \in E_m$ illustrates a known Szegö theorem from 1925.

In addition we shall also discuss some convex properties which, in particular, implies that HCP condition is a consequence of Markov's property and nonpluripolarity of a compact set in $\mathbb{C}^N$.

L_p Markov exponent of certain domains with cusps

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Abstract: We say that a compact set $\emptyset \neq E \subset \mathbb{R}^m$ satisfies L_p Markov type inequality (or: is a L_p Markov set) if there exist $\kappa, C > 0$ such that, for each polynomial $P \in \mathcal{P}(\mathbb{R}^m)$ and each $\alpha \in \mathbb{N}_0^m$,

$$\|D^\alpha P\|_{L_p(E)} \leq (C(\deg P)^\kappa)^{|\alpha|} \|P\|_{L_p(E)}, \tag{1}$$

where $D^\alpha P = \frac{\partial^{|\alpha|} P}{\partial x_1^{\alpha_1} \ldots \partial x_m^{\alpha_m}}$ and $|\alpha| = \alpha_1 + \cdots + \alpha_m$.
In this talk we shall consider the following problem:

For a given L_p Markov set E determine $\mu_p(E) := \inf\{\kappa : E$ satisfies (1)$\}$.

The quantity $\mu_p(E)$ is called L_p Markov exponent and was first considered by Baran and Pleśniak in [2] for $p = \infty$. This is related to the linear extension operator for C^∞ functions with restricted growth of derivatives (see [6,7]). For any compact set E in $\mathbb{R}^m$ we have $\mu_p(E) \geq 2$. If E is a fat convex subset of $\mathbb{R}^m$, then $\mu_p(E) = 2$. It is known that L_∞ Markov exponent, for $Lip\gamma$, $0 < \gamma < 1$ cuspidal domains in $\mathbb{R}^m$ is equal to $\frac{2}{\gamma}$ (see for instance, [3], [1], [4]). If $K \subset \mathbb{R}^m$ is a $Lip\gamma$, $0 < \gamma < 1$ cuspidal piecewise graph domain such that it is imbedded in an affine image of the l_γ ball having one of its vertices on the boundary ∂K of K, then $\mu_p(E) = \frac{2}{\gamma}$ for $1 \leq p < \infty$ (see [5]). Our goal is to establish L_p Markov exponent of the following domains $\Psi_k := \{(x, y) \in \mathbb{R}^2 : |x| \leq 1, 0 \leq y \leq x^{2k}\}$, $\Upsilon_k := \{(x, y) \in \mathbb{R}^2 : |x| \leq 1, 0 \leq y \operatorname{sgn}(x) \leq |x|^{2k+1}\}$ and $\Lambda_k := \{(x, y) \in \mathbb{R}^2 : 0 \leq x \leq 1, ax^k \leq y \leq bx^k\}$, $k \in \mathbb{N}$, $0 < a < b$. More precisely, we show that $\mu_p(\Psi_k) = \mu_p(\Lambda_k) = 2k$ and $\mu_p(\Upsilon_k) = 2k + 1$ for every $k \in \mathbb{N}$, $1 \leq p < \infty$. Since none of the domains Ψ_k, Υ_k and Λ_k is cuspidal piecewise graph domain, the above results cannot be obtained using the methods of [5]. In particular, Λ_k has a cusp at the origin that cannot be connected to the interior of Λ_k by a straight line. However, these results are known in case of supremum norm (see [4]).

References

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Admissible meshes on algebraic sets

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Abstract: We construct polynomial weakly admissible meshes on compact subsets of algebraic hypersurfaces in $\mathbb{C}^N$. They are preimages by a projection of meshes on compacts in $\mathbb{C}^{N-1}$. These meshes are optimal in some cases. The problem is much more complicated for an arbitrary algebraic variety and we present only partial results for algebraic sets of codimension greater than one.

Based on a joint paper with Agnieszka Kowalska.

Multidimensional pseudo-Leja sequences

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Abstract: As part of my PhD project, I am interested in the selection of interpolation nodes for the approximation of holomorphic functions. The one-dimensional pseudo-Leja sequences introduced by Białas-Cież and Calvi(2012), as an alternative to Leja sequences, provide us with good interpolation nodes for approximating analytic functions. Here we first propose a definition of multidimensional pseudo-Leja sequences associated to a compact set K of the complex space $\mathbb{C}^n$ which is consistent with the one-dimensional one. We show that these sequences can be used to calculate the transfinite diameter of K. Subsequently, we show that it is possible to construct a multidimensional pseudo-Leja sequence by considering the intertwining sequences of the one-dimensional pseudo-Leja sequences.

References

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The extremal function of a real convex body

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Abstract: I describe a formula for the extremal function of a convex polytope in terms of extremal functions of supporting simplices and strips of the polytope. I give the basic steps in the derivation of the formula, which use convex geometry and pluripotential theory. The square mapping (used by Baran to relate the extremal functions of a ball and simplex) also plays an important part in the derivation.