

# Stable and Chaotic Motions in the Planetary Problem

Osservatorio Astrofisico di Asiago

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## Lectures

Giancarlo Benettin, University of Padua, Italy

*title: TBA*

Abstract: TBA

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*Lecture 1. A review of cometary orbital families*

Comets are nowadays believed to be the most pristine objects left from the Solar System formation and are divided in several classes, or families, according to their orbital properties. In this lesson the physical nature of comets and the properties of the different classes will be reviewed also in terms of the origin place in the Solar System and the dynamical evolution.

*Lecture 2. The ESA/Rosetta mission interplanetary and escort orbits*

The milestone ESA/Rosetta mission was designed to follow and study the Jupiter-family comet 67P/Churyumov–Gerasimenko for almost two years around the last perihelion passage. Without a propulsion system, the spacecraft was forced to perform a complex interplanetary trajectory which lasted for 10 years and involved three gravity assists with our planet and one with Mars, while passing two times in the main asteroid belt. Once arrived at the comet, the two years escorting of the comet was performed with continuous attitude correction maneuvers along non-gravitationally bound

orbits, pushing to the limits the capabilities of the spacecraft itself. The engineering solutions to navigate to and along with a small active body such as a comet are in this lesson portrayed.

**Jerome Daquin**, University of Padua

*Lunisolar resonant dynamics*

This short lecture will discuss some dynamical properties related to the (secular) energy function of a restricted Earth-Satellite-Moon-Sun system with an oblate primary. In particular, we will characterise the lunisolar resonant web and show that Chirikov's criterion, so often applied in connected fields, underlies the chaos representation. The extent and fine geometry of chaos are then portrayed using dynamical indicators by looking to the destruction of KAM curves. The approach turns out to be useful in modern applications to remedy the Space Debris problem.

**Vadim Kaloshin**, University of Maryland

*Arnold diffusion across strong double resonance*

Arnold's conjecture states that a generic nearly integrable Hamiltonian system possesses diffusing orbits, i.e. those where action changes considerably. In a joint work with K. Zhang, following ideas of Mather, we prove this conjecture for smooth perturbations of convex integrable systems of 2.5 degrees of freedom. Diffusing orbits should go through two different regimes: single resonance and double resonance. The single resonant regime was handled in the joint work with Bernard and K. Zhang. During the lectures we discuss the other regime: double resonance. We present an outline of the proof of existence of diffusing orbits there. The proof has several ingredients:  $c$ -equivalence and variational lambda lemma (by Mather-Bernard), construction of 'kissing' cylinders at the double resonance, variational diffusion along such cylinders and the jump lemma.

**Fiorangela La Forgia**, University of Padua

*Dynamics of Gas and Dust of Comets*

Comets are small transient and primitive objects of the Solar System, which enclose very important information on the pristine state of our planetary environment. Their physical structure comprises a nucleus, an expanding

coma and several tails. In order to interpret the complex observations of comets and understand their physical states, models for the kinematics of gas species and dynamics of dust in the coma and tails are required. In this lesson the gas kinetics and dust dynamics of comets are reviewed, from both theoretical and observational perspectives, which are important for understanding the wide variety of physical conditions that are encountered in studying comets.

**Francesco Marzari**, University of Padua

*Extrasolar planets dynamics*

The different dynamical mechanisms which may affect the evolution of a system of planets will be discussed, in particular those responsible for small semi-major axes, high eccentricity and inclination. I will focus on 1) planet migration by interaction with the proto-stellar disk of a single planet or of a pair of planets in resonance, 2) planet-planet scattering followed by tidal circularization and potential Kozai contribution to the inclination evolution, 3) resonance trapping in presence of the gaseous disk and planetesimal scattering. I will also explore how planets can shape debris disks which may be used as indirect evidence of the planet presence.

**Richard Moeckel**, University of Minnesota

*Blowing up the N-body problem*

From the mathematical point of view, one of the special features of the Newtonian N-body problem which makes it so interesting and so difficult is the non-compactness of the phase space. The bodies can fly off to infinity or reach the excluded set of collision singularities. McGehee's blowup method provides a partial compactification which allows one to study solutions near total collision (all bodies collide one point) or parabolic infinity (all bodies tend to infinity with zero limiting velocity). Then, using mostly qualitative geometrical reasoning, one can prove existence of a variety of interesting solutions. My lectures will introduce McGehee's coordinates and discuss several applications with emphasis on low-dimensional cases where one can visualize the results with pictures. Even familiar classical results can be illuminated by blowing things up. Some of the topics to be covered: the blown-up two-body problem – collisions and parabolic orbits, the shape sphere for the planar three-body problem, spiraling stable and unstable manifolds, solutions bi-asymptotic to triple collision, symbolic dynamics and chaos near triple collision, realizing syzygy sequences and the Lagrangian

property of manifolds of parabolic solutions.

**Alessandro Morbidelli**, Observatoire de la Côte d'Azur

*Dynamical Evolution of Planetary Systems*

Planetary systems can evolve dynamically even after the full growth of the planets themselves. There is actually circumstantial evidence that most planetary systems become unstable after the disappearance of gas from the protoplanetary disk. These instabilities can be due to the original system being too crowded and too closely packed or to external perturbations such as tides, planetesimal scattering, or torques from distant stellar companions. The Solar System was not exceptional in this sense. In its inner part, a crowded system of planetary embryos became unstable, leading to a series of mutual impacts that built the terrestrial planets on a timescale of 100My. In its outer part, the giant planets became temporarily unstable and their orbital configuration expanded under the effect of mutual encounters. A planet might have been ejected in this phase. Thus, the orbital distributions of planetary systems that we observe today, both solar and extrasolar ones, can be different from the those emerging from the formation process and it is important to consider possible long-term evolutionary effects to connect the two.

**Gabriella Pinzari**, University of Padua

*Arnold's properly-degenerate KAM theory and beyond*

The aim of this lecture is to review and revisit the main ideas of KAM theory developed by V.I. Arnold in 1963 for properly-degenerate systems. The necessity of complementing, in the case of application to problems arising from celestial mechanics, analytic and geometric tools will be briefly discussed.

**Tere-M. Seara**, Universitat Politècnica de Catalunya

*Arnold diffusion: old and new geometric methods*

The aim of this lectures is to present how geometric methods can be used to prove the existence of Arnold diffusion in nearly-integrable Hamiltonian systems. We will first give an overview of these methods: the use of normally hyperbolic invariant manifolds, the study of the invariant objects by the inner dynamics contained in them, the transversal intersection of their sta-

ble and unstable manifolds, and the scattering map. Finally we will present a new mechanism that uses very few information about the inner dynamics and that can be used in several settings under explicit and checkable conditions to obtain orbits with big changes in actions.