Perspectives in Hamiltonian Dynamics

Centro Culturale Don Orione, Venezia, June 18-22, 2018
University of Padua
Funded by the European Research Council

Program

Sunday:

Arrival of the participants at the Centro Culturale Don Orione Artigianelli, Venice

***Monday**:*

Sala Goldoni

Registration (8.30-9.00)

Opening (9.00-9.30)

***V. Kaloshin** (9.30-10.30)*

Coffee--break (10.30-11.00)

***M. Berti** (11.00-12.00)*
***C. Efthymiopoulos** (12.00-13.00)*

Lunch (13:15-)*

***A. Celletti** (15.30-16:30)*
***S.Terracini** (16.30-17:30)*

Coffee--break (17.30-18.00)

Sala Goldoni	***Sala Vivaldi***
***A. Clarke** (18.00-18.30)*	***S. Barbieri** (18.00-18.30)*
***N. Duignan** (18.30-19.00)*	***A. Pousse** (18.30-19.00)*

***Tuesday**:*

Sala Goldoni

***S. Kuksin** (8.30-9.30)*
***A. Chenciner** (9.30-10.30)*

Coffee--break (10.30-11.00)

***B. Fayad** (11.00-12.00)*
***D. Bambusi** (12.00-13.00)*

Lunch (13:15-)

***K. Zhang** (15.30-16:30)*
***A. Sorrentino** (16.30-17:30)*

Coffee--break (17.30-18.00)

Sala Goldoni

***F. Mogavero** (18.00-18.30)*
***S. Di Ruzza** (18.30-19.00)*

Sala Vivaldi

***I. Gkolias** (18.00-18.30)*
***M. Orieux** (18.30-19.00)*

***Wednesday**:*

Sala Goldoni

***A. Delshams** (8.30-9.30)*
***M. Gidea** (9.30-10.30)*

Coffee--break (10.30-11.00)

***G. Pinzari** (11.00-12.00)*
***E. Lega** (12.00-13.00)*

Lunch (13:15-)

***B. Khesin** (15.30-16:30)*
***H. Dullin** (16.30-17:30)*

Coffee--break (17.30-18.00)

Sala Goldoni

***R. Paez** (18.00-18.30)*
***S. Marò** (18.30-19.00)*

Sala Vivaldi

***M. M. Castro** (18.00-18.30)*
***A. Boscaggin** (18.30-19.00)*

***Thursday**:*

Sala Goldoni

***T. Seara** (8.30-9.30)*
***J-P. Marco** (9.30-10.30)*

Coffee--break (10.30-11.00)

***D. Treschev** (11.00-12.00)*
***G. Gronchi** (12.00-13.00)*

Lunch (13:15-)

Sala Goldoni	***Sala Vivaldi***
***M. Guardia** (15.30-16:30)*	
***F. Giuliani** (16.30-17.00)*	***V. Ortega** (16.30-17.00)*
***A. Vieiro** (17.00-17.30)*	***A-S. Libert** (17.00-17.30)*
Coffee--break (17.30-18.00)	
***A. Portaluri** (18.00-18.30)*	***M. Sansottera** (18.00-18.30)*
***Gouwei Yu** (18.30-19.00)*	***T. Penati** (18.30-19.00)*

***Friday**:*

Sala Goldoni

***J. Laskar** (8.30-9.30)*
***D. Turaev** (9.30-10.30)*

Coffee--break (10.30-11.00)

***E. Valdinoci** (11.00-12.00)*
***R. Moeckel** (12.00-13.00)*

Lunch (13:15-)

**The lunch is offered to all participants at the Ristorante San Trovaso*

the organizing committee

Plenary Lectures

Dario Bambusi, University of Milano, Milano, Italy

Some large probability normal forms results for Hamiltonian systems: from FPU to NLS

The so called FPU system is a chain composed by a large number of particles interacting through nonlinear springs. This is the simplest possible model of a solid and has been used (since the numerical experiment by Fermi Pasta and Ulam) to try to understand if the ergodicity properties needed in order to develop statistical mechanics are actually verified in a realistic model. I will present a result which is valid with large probability with respect to the Gibbs measure and ensures that, provided the temperature (the inverse of the parameter β in the measure) is small enough, then the energy of each packet of normal modes that one can form remains approximatively constant for times of orderβ. This shows in particular that the dynamics has strong regularity propeerties for times increasing quite fast as the temperature goes to zero. Result in collaboration with A. Carati and A. Maiocchi. Technically the result is based on some tools that have been developed by Maiocchi and Carati in order to study the dynamics of chains of particles in Gibbs measure. The same tools can be adapted to the case of PDEs. In particular I will present an averaging type result for the defocousing NLS in one space dimensions (for which it is known that the Gibbs measure exists and is invariant) ensuring that all the actions are almost surely approximately integral of motion for long times. Result in collaboration with A. Maiocchi and L. Turri.

Massimiliano Berti, Scuola Internazionale di Studi Superiori Avanzati, Trieste, Italy

Dynamics of Water Waves

In this talk I shall present recent results about the complex dynamics of the water waves equations of a bi-dimensional fluid under the action of gravity and/or capillary forces, with space periodic boundary condition, that, since Zakharov it is known to be an infinite dimensional Hamiltonian system. We shall discuss both Birkhoff normal form long time existence results as well as new KAM results about the bifurcation of small amplitude quasi-periodic solutions. Major difficulties are the quasilinear nature of the water waves equations.

Alessandra Celletti, Università Tor Vergata, Rome, Italy

Resonances and chaos in space debris

The study of resonances and chaos within space debris dynamics is of paramount importance for the analysis of the global behavior of small objects orbi-

ting around the Earth. In particular, it is important to understand the location of the equilibria, the existence of regular or chaotic regions, and especially their behavior as some control parameters are varied. These studies might allow to develop mitigation, maintenance and control strategies based on mathematical investigations. Numerical and analytical techniques can be used to investigate special resonances affecting the dynamics of space debris in different regions of the space around our planet. This allows to discover different phenomena, like the splitting and overlapping of resonances, bifurcations, and chaos occurrence as the orbital elements are varied. A study through a suitable normal form theory allows to obtain results about geostationary objects with interesting applications at high area-to-mass ratio space debris.
This talk refers to works in collaboration with C. Gales, C. Efthymiopoulos, F. Gachet, G. Pucacco.

Alain Chenciner, ASD, IMCCE, Observatoire de Paris, Paris, France

N-body relative equilibria in higher dimensions

If one allows the dimension of the ambient Euclidean space to be greater than 3, the family of n-body configurations which, when submitted to Newtonian or similar attraction, admit a relative equilibrium motion (the "balanced" configurations) becomes much richer. Also, a given balanced configuration admits a variety of relative equilibria, namely one for each choice of a hermitian structure on the space where the motion really takes place; in general, if the configuration is not central, such relative equilibria are quasi-periodic. I shall discuss several problems, like the one of deciding what is the smallest dimension in which a given configuration admits a relative equilibrium motion, or when bifurcations from the periodic relative equilibrium of a central configuration may bifurcate into a family of quasi–periodic relative equilibria of balanced configurations.

Amadeu Delshams, Universitat Politécnica de Catalunya, Barcelona, Spain

Exponentially small splitting of separatrices associated to 3D whiskered tori with cubic frequencies

The measure of the exponentially small splitting of separatrices in nearly-integrable Hamiltonian systems is crucial for proving the existence of chaotic motions and global instability. This is a difficult problem for whiskered tori with more than two frequencies, since then there is no standard theory of continued fractions.
In this talk we consider the splitting of invariant manifolds of whiskered (hyperbolic) tori with three frequencies in a nearly-integrable Hamiltonian system, whose hyperbolic part is given by a pendulum. We consider a 3–dimensional torus with a fast frequency vector $\omega/\sqrt{\varepsilon}$, with $\omega = (1, \Omega, \Omega^2)$ where Ω is a cubic irrational number whose two conjugates are complex (for instance, the real root of $z^3 + z - 1$). Applying the Poincaré–Melnikov method, we carry out a careful study of the dominant harmonics of the Melnikov function. This allows us to provide an asymptotic estimate for the maximal splitting distance, which is exponentially small in ε, and valid for all sufficiently small values of ε. The

function in the exponent turns out to be quasi–periodic with respect to $\log(\varepsilon)$, and can be explicitly constructed from the resonance properties the frequency vector ω. In this way, we emphasize the strong dependence of our results on the arithmetic properties of the frequencies. This is a joint work with Marina Gonchenko and Pere Gutiérrez.

Holger Dullin, University of Sydney, Australia

Monodromy in the Kepler Problem

What could possibly be said about the Kepler Problem that is new? It is well known that this superintegrable system can be separated in different coordinate systems, and each such separation defines a distinct Liouville integrable system. We show that for separation of the spatial system in prolate spheroidal coordinates the resulting integrable system has Hamiltonian monodromy. This is a semi-toric system with two degrees of freedom on $S^2 \times S^2$ that is obtained by symplectic reduction of the S^1 action generated by the Kepler Hamiltonian. Analogous results are obtained for the corresponding quantum integrable system, where the eigenfunctions are spheroidal harmonics. Similar analysis can be done for many prominent superintegrable systems, for example the harmonic oscillator, the free particle or the geodesic flow on the sphere. In all these cases the resulting reduced systems correspond to well known special functions, but the quantum monodromy in their joint spectrum is reported here for the first time.
Joint work with Holger Waalkens, University of Groningen
Phys. Rev. Lett. 120, 020507 (2018)

Christos Efthymiopoulos, Research Center for Astronomy and Applied Mathematics, Academy of Athens, Athens, Greece

Methods and Applications in Manifold Dynamics: from molecules to Space and galaxies

Manifold dynamics near unstable invariant sets in the phase space of nonlinear Hamiltonian systems is a rapidly expanding domain of research. A variety of applications have been presented in problems related to physical scales ranging from the molecular to the galactic one. The talk will review recent progress in this topic. In particular, we will discuss:
i) Perturbative (e.g. normal form or parametrization) as compared to numerical (e.g. propagation or FLI-based) methods of computation of manifolds and their neighborhood,
ii) applications in escape dynamics (e.g. reaction or dissociation rates in molecules, planetesimal growth rates, tidal streams and mass loss in satellite galaxies), and
iii) applications of recurrent manifold dynamics (e.g. light propagation in nonlinear media, space-manifold dynamics, jumping asteroids in the solar system, and spiral structure in galaxies).
As regards methods of computation, we will argue on the efficiency of perturba-

tive methods, which over the years become more and more competitive against purely numerical ones. As regards the applications, we will focus on the unifying aspects and computational benefits offered by manifold-based dynamical theories, which empowers common interpretation of a variety of apparently disconnected (in terms of context and/or scale) physical phenomena.

Bassam Fayad, Institut de Mathématiques de Jussieu–Paris Rive Gauche, Paris, France

Unstable elliptic fixed points in Hamiltonian dynamics

We introduce a new diffusion mechanism from the neighborhood of elliptic equilibria for Hamiltonian flows in three or more degrees of freedom. Using this mechanism, we obtain the first examples of real analytic Hamiltonians that have a Lyapunov unstable non-resonant elliptic equilibrium.

Marian Gidea, Yeshiva University, New York, USA

Arnold Diffusion in the Elliptic Restricted Three-Body Problem: A Computer Assisted Proof

We consider the planar elliptic restricted three-body problem, viewed as a perturbation of the planar circular restricted three-body problem. We use realistic parameters for the masses of the primaries. We show that there exist orbits along which the energy changes by some constant that is independent of the perturbation parameter, as well as orbits for which the energy undergoes symbolic dynamics. We provide quantitative estimates on diffusing orbits: diffusion speed, Hausdorff dimension, and range of perturbation parameter for which orbits change energy by a given constant. This study is motivated by the Arnold diffusion problem. Our approach combines geometric and topological methods with computer assisted proofs. This is based on joint works with M. Capinski and R. de la Llave.

Giovanni Gronchi, Università di Pisa, Pisa, Italy

On the existence of connecting orbits for critical values of the energy

We consider an open connected set $\mathcal{A}$ and a smooth potential U which is positive in A and vanishes on the boundary. We study the existence of orbits of the mechanical system

$$\frac{du}{dt} = \nabla U(u)$$

that connect different components of the boundary of $\mathcal{A}$ and lie on the zero level of the energy. We allow that the boundary of $\mathcal{A}$ contains a finite number of critical points of U. The case of symmetric potential is also considered.
This is a joint work with Giorgio Fusco and Matteo Novaga, appeared in J. Differential Equations 263 (2017) pp. 8848–8872

Marcel Guardia, Universitat Politécnica de Catalunya, Barcelona, Spain

Growth of Sobolev norms for the cubic NLS near 1D quasi-periodic solutions

Consider the defocusing cubic Schrödinger equation defined in the 2 dimensional torus. It has as a subsystem the one dimension cubic NLS (just considering solutions depending on one variable). The 1D equation is integrable and admits global action angle coordinates. Therefore, all its solutions are either periodic, quasi-periodic or almost-periodic. Consider one of the finite dimensional quasi–periodic invariant tori that the 1D equation possesses. Under certain assumptions on the torus (smallness, Diophantine frequency), we show that there exist solutions of the 2D equation which start arbitrarily close to this invariant torus in the H^s topology (with $0 < s < 1$) and whose H^s Sobolev norm can grow by any given factor. This is a joint work with Z. Hani, E. Haus, A. Maspero and M. Procesi.

Vadim Kaloshin, University of Maryland, Maryland, USA

Can you hear the shape of a drum and deformational spectral rigidity of planar domain?

M. Kac popularized the question 'Can you hear the shape of a drum?'. Mathematically, consider a bounded planar domain and the associated Dirichlet problem. The set of 's such that this equation has a solution, is called the Laplace spectrum of . Does Laplace spectrum determines? In general, the answer is negative.
Consider the billiard problem inside. Call the length spectrum the closure of the set of perimeters of all periodic orbits of the billiard. Due to deep properties of the wave trace function, generically, the Laplace spectrum determines the length spectrum. We show that a generic axis symmetric planar domain with sufficiently smooth boundary is dynamically spectrally rigid, i.e. can't be deformed without changing the length spectrum. This partially answers a question of P. Sarnak.
Arguably the most important ingredient of the proof is a nearly integrable Hamiltonian structure of dynamics of nearly glancing orbits. This is based on two joint works with J. De Simoi, Q. Wei and J. De Simoi, A. Figalli.

Boris Khesin, University of Toronto, Toronto, Canada

Geometric and Hamiltonian hydrodynamics via Madelung transform

We introduce a geometric framework to study Newton's equations on infinite-dimensional configuration spaces of diffeomorphisms and smooth probability densities. It turns out that several important PDEs of hydrodynamical origin can be described in this framework in a natural way. In particular, the so-called

Madelung transform between the Schrödinger-type equations on wave functions and Newton's equations on densities turns out to be a Kähler map between the corresponding phase spaces, equipped with the Fubini-Study and Fisher-Rao information metrics. This is a joint work with G.Misiolek and K.Modin.

Sergei Kuksin, Université Paris 7 Diderot, Paris, France

Long time behaviour for solutions of damped and driven Hamiltonian PDEs

Consider a Hamiltonian PDE on a torus. Long time behaviour of its solutions is hopelessly complicated problem, related with the ergodic hypothesis. Now let us add to the system a dissipation, given by a positive degree of minus-Laplacian, and a random force, which is bounded, arbitrarily small and affects only a few Fourier modes. I will show that if the damped/driven equation is well posed and the nonlinearity is non-degenerate in certain cense, then for this perturbed system the long-time behaviour of solutions is described by a certain unique measure in the function space (independent from the initial data). The proof uses the Doeblin method and KAM.
This is a joint work with A.Shirikyan and V.Nersesyan, arXiv:1802.03250v1

Jacques Laskar, IMCCE - Observatoire de Paris, Paris, France

AMD-stability and the classification of planetary systems

In a planetary system, the AMD (Angular Momentum Deficit) is the difference between the planar circular angular momentum and the total angular momentum. This quantity is conserved between collisions in the average system, and decreases during collisions.
This leads to the concept of AMD-stability. A planetary system is AMD-stable if the AMD in the system is not sufficient to allow collisions. The advantage of this notion is that it becomes possible to verify very quickly whether a newly discovered planetary system is stable or potentially unstable, without any numerical integration of the equations of motion. These principles have been applied to the 131 multiple planetary systems of the exoplanet.eu database whose orbital elements are sufficiently well determined (Laskar and Petit, 2017a).
AMD-stability, based on the secular evolution, addresses to long time stability, in absence of mean motion resonances. On the other hand, criterions for short term stability have been established on the basis of Hill radius (Marchal & Bozis 1982; Gladman 1993; Pu & Wu 2015) or on the overlap of mean motion resonances (Wisdom 1980; Duncan et al. 1989; Mustill & Wyatt 2012; Deck et al. 2013). Both long and short time scales can be combined owing some modification of the AMD-stability criterion (Petit, Laskar & Boué, 2017).
Ref: Laskar, J. and Petit, A.C., 2017, AMD-stability and the classification of planetary systems, A&A, 605, A72 Petit, A.C. Laskar, J. and Boué, G., 2017, AMD-stability in presence of first order mean motion resonances, A&A, 607, A35

Elena Lega, Laboratoire Lagrange CNRS, Université Côte d'Azur, Observatoire de la Côte d'Azur, Nice, France

Computation of the hypertube manifolds in the spatial circular restricted three-body problem with chaos indicators and guided visualisations

The circular restricted three-body problem has five relative equilibria L_1, L_2, ..., L_5. The invariant stable-unstable manifolds of the center manifolds originating at the partially hyperbolic equilibria L_1,L_2 have been identified as the separatrices for the motions which transit between the regions of the phase–space which are internal or external with respect to the two massive bodies.
While the stable and unstable manifolds of the planar problem have been extensively studied both theoretically and numerically, the spatial case has not been as deeply investigated.
This talk is devoted to the global computation of these manifolds in the spatial case with a suitable finite time chaos indicator. We introduce finite time chaos indicators and show examples of computation of the standard fast Lyapunov indicator for the restricted three-body problem.
For this specific case, the definition of the chaos indicator is not trivial, since the mandatory use of the regularizing Kustaanheimo-Stiefel variables may introduce discontinuities in the finite time chaos indicators. From the study of such discontinuities, we define geometric chaos indicators which are globally defined and smooth, and whose ridges sharply approximate the stable and unstable manifolds of the center manifolds of L_1, L_2. We illustrate the method for the Sun-Jupiter mass ratio, and represent the topology of the asymptotic manifolds using sections and three-dimensional representations.

Jean–Pierre Marco, Université Pierre et Marie Curie, Paris, France

Symplectic geometry, normally hyperbolic invariant cylinders and KAM *theorem with vanishing torsion*

In this talk we will give a general presentation of the symplectic properties of normally hyperbolic manifolds invariant under symplectic diffeomorphisms and show how they yield simple regularity properties for their stable and unstable foliations. This is turn enables one to get efficient "geometric normal forms (without any iterative scheme) for the diffeomorphism in the neighborhood of the normally hyperbolic manifold. We then use this study to prove the existence of compact invariant cylinders (diffeomorphic to $\mathrm{T}^2 \times [0, 1]$) located near double resonances in perturbations of angle-action hamiltonian systems on $T^*\mathbb{T}^3$. The boundaries of these cylinders turn out to be hyperbolic KAM tori, whose existence is deduced from an adapted KAM theorem with vanishing torsion, taking advantage of the previous normal forms.

Richard Moeckel, University of Minnesota, USA

Relative Equilibria of Gravitationally Interacting Rigid Bodies

Consider the motion of N solid bodies interacting via Newtonian gravitational forces. Dissipative forces, such as tidal friction, may cause the total energy of the system to decease while angular momentum remains constant. This leads to the problem: find the minima of the energy on a submanifold of fixed angular momentum in phase space. The critical points of the energy correspond to relative equilibrium motions where all the bodies are phase locked and the whole system just rotates rigidly around the center of mass. Pluto and its moon Charon provide an approximate example of this phenomenon with N=2 bodies. The first part of the talk presents a theorem that for $N = 3$ or more bodies, local minima of energy do not exist − all of the relative equilibria are saddle points. Presumably, this means that such rigid, phase locked motions do not occur for more than 2 bodies. The second part of the talk is estimating the number of relative equilibria for N=2 using Morse theory.

Gabriella Pinzari, University of Padua, Padua, Italy

A new quasi–integral in the three–body problem

Whenever a dynamical system appears as a small perturbation of a Liouville–Arnold integrable system, it is possible to identify functions that, generically, remain confined closely to their initial values over exponentially–long times. Those functions were named "quasi–integrals" by N.N. Nekhorossev. The three–body problem has a quasi–integrable structure which, as H. Poincaré and V.I. Arnold highlighted, strongly depends on the choice of the masses. We shall show that, if the masses have three well separated sizes, it is possible to identify a "new" quasi–integral in the sense of Nekhorossev. Here "new" means that this function is a combination of known quasi–integrals, whose slow variations are not independent, but form the level sets of the new function. Such level sets are smooth, connected and compact in any region of phase space where collisions do not occur, and change their topology in correspondence of collisions. As an application, using KAM theory and a suitable normal for theory with a–periodic coordinates, we shall show that, in the planar three–body problem, in any region of phase space arbitrarily close to collisional orbits between the two smaller bodies, there exist quasi–periodic motions with three independent frequencies. The key of the proof relies on the use of the two–centre problem as Liouville–Arnold integrable reference system. Possible scenarios of Arnold instability will be also highlighted. This talk is based on a recent work, mostly still in progress, of the author [arXiv:1607.03056], [arXiv:1702.03680], [arXiv:1710.02689].

Tere M.–Seara, Dept. de Matemàtica Aplicada I, U. Politècnica de Catalunya, Barcelona, Spain

A General Mechanism of Diffusion in Hamiltonian Systems

In this talk we first present a general shadowing result for normally hyperbolic invariant manifolds. The result does not need any knowledge of the (inner) dynamics restricted to the normally hyperbolic manifold.
We apply this result to establish the existence of diffusing orbits in a large class of nearly integrable Hamiltonian systems. Our approach relies on successive applications of the so called 'scattering map' along homoclinic orbits to a normally hyperbolic invariant manifold. The main idea is that we can closely follow any path of the scattering map. This gives the existence of diffusing orbits.
The method applies to perturbed integrable Hamiltonians of arbitrary degrees of freedom (not necessarily convex) which present some hyperbolicity without any assumption about the inner Dynamics. This is joint work with Marian Gidea (Yeshiva U., Dept. of Mathematical Sciences, New York, USA) and Rafael de La Llave (School of Mathematics, Georgia Institute of Technology, Atlanta, USA).

Alfonso Sorrentino, Università Tor Vergata, Rome, Italy

On the Birkhoff conjecture for convex billiards

A mathematical billiard is a system describing the inertial motion of a point mass inside a domain, with elastic reflections at the boundary. This simple model has been first proposed by G.D. Birkhoff as a mathematical playground where it the formal side, usually so formidable in dynamics, almost completely disappears and only the interesting qualitative questions need to be considered. Since then, billiards have captured much attention in many different contexts, becoming a very popular subject of investigation. Despite their apparently simple local dynamics, their qualitative dynamical properties are extremely non-local; this global influence on the dynamics translates into several intriguing rigidity phenomena, which are at the basis of unanswered questions and conjectures. In this talk I shall focus on some of these questions. In particular, I shall describe some recent results related to the classification of integrable billiards, also known as the Birkhoff conjecture. This is based on joint works with G.Huang and V. Kaloshin.

Susanna Terracini, Università di Torino, Torino, Italy

Title: TBA

Abstract: TBA

Dmitri Treschev, Steklov Mathematical Institute of Russian Academy of Sciences, Russia

Travelling waves, Fermi-Pasta-Ulam lattices

We consider a classical mechanical systems composed of an infinite number of discrete particles on a line. Each particle is assumed to interact with the nearest left and right neighbors only. We construct travelling waves in the system assuming that the potential has a singularity at zero. The waves appear near the hard ball limit. We also discuss further results and problems.

Dmitri Turaev, Imperial College, London, United Kingdom

On Herman's positive metric entropy conjecture

Abstract: TBA

Enrico Valdinoci, University of Milano, Milano, Italy

Chaotic orbits for nonlocal equations and applications to atom dislocation dynamics in crystals

We consider a nonlocal equation driven by a perturbed periodic potential. We construct multibump solutions that connect one integer point to another one in a prescribed way. In particular, heteroclinic, homoclinic and chaotic trajectories are constructed. This result regarding symbolic dynamics in a fractional framework is part of a study of the Peierls-Nabarro model for crystal dislocations. The associated evolution equation can be studied in the mesoscopic and macroscopic limit. Namely, the dislocation function has the tendency to concentrate at single points of the crystal, where the size of the slip coincides with the natural periodicity of the medium. These dislocation points evolve according to the external stress and an interior potential, which can be either repulsive or attractive, depending on the relative orientations of the dislocations. For opposite orientations, collisions occur, after which the system relaxes exponentially fast.

Ke Zhang, University of Toronto, Toronto, Canada

Diffusion limit for slow-fast standard maps

We discuss the following family of slow-fast standard maps on the cylinder: $F_\epsilon(x,y) = (x + \epsilon^{-a}\sin(2\pi x) + \epsilon^{-1-a}y, y + \epsilon\sin(2\pi x))$ with a parameter $a > 0$. This map is the composition of a ϵ-size tilt map, and a ϵ^{-1-a}-size twist map. The case $a = 1$ can be seen as a toy model for scattering by weak resonance in slow-fast Hamiltonian systems. Assume that a is large enough ($a > 8$), we prove that starting from a (Lebesgue) random initial condition, the ϵ^{-2}th iterate of the slow variable converges weakly to a Gaussian random variable as $\epsilon \to 0$.

This is a joint work with Alex Blumenthal and Jacopo De Simoi.

Contributed Lectures

Santiago Barbieri, University of Padua, Italy

Sharp Nekhoroshev estimates for the three-body problem around a periodic torus

Nekhoroshev theorem on quasi-integrable hamiltonian systems has been widely studied in the past decades and many are its applications in celestial mechanics. However, when considering Nekhoroshev stability for a convex system around a periodic torus, the available proofs contained non-sharp and often implicit estimates. Moreover, a rigorous estimate on the analyticity domain for the three-body planetary hamiltonian lacked, thus leaving uncertainty when considering explicit values for the analyticity widths. In this work, we tried to sharpen as much as possible all the quantities that appeared in the proof of Nekhoroshev theorem and we made use of a recent rigorous result by T. Castan on the analyticity domain for the three-body planetary problem. Moreover, a suitable application to the restricted three-body problem was also considered. Indeed, such celestial systems are simple enough to allow for a disentanglement of the physical thresholds for the applicability of the theorem, so that the aim of this study is to enlight the role that complex singularities, the number of averaging steps and the use of Cauchy estimates play in ensuring Nekhoroshev stability, as well as to point out the role that the same quantities play in preventing the magnitude of the perturbation to reach a physical value.

Alberto Boscaggin, University of Torino, Italy

Periodic solutions to perturbed Kepler problems

As well known (by third Kepler's law) the Kepler problem has many periodic solutions with minimal period T (for any given $T > 0$). We will try to understand how many of them survive after a T-periodic external perturbation preserving the Newtonian structure of the equation. In doing this, we will be naturally led to the concept of generalized solution and to the theory of regularization of collisions in Celestial Mechanics.
Joint work with R. Ortega and L. Zhao. Periodic solutions and regularization of a Kepler problem with time-dependent perturbation, to appear on Trans. Amer. Math. Soc.

Andrew Clarke, Imperial College, United Kingdom

Arnold diffusion for a-priori chaotic category of analytic convex billiards (joint work with D. Turaev)

Let Γ be an analytic, closed, and strictly convex hypersurface of $\mathbb{R}^d$ where

$d \geq 3$, and suppose the geodesic flow on $T\Gamma$ has a nontrivial hyperbolic set. Let $(x(t), u(t))$ denote a trajectory of the geodesic flow, where $x(t) \in \Gamma$, and $u(t) \in T_{x(t)}\Gamma$. Then $||u(t)||$ (which we assume to be ≤ 1) is a constant of motion. The billiard map sends a point $x \in \Gamma$ and an inward pointing velocity vector $v = u + \sqrt{1-u^2}\hat{n}(x)$ (where $u \in T_x\Gamma$, and $\hat{n}(x)$ denotes the inward pointing unit normal) to the next point of intersection with Γ in the direction of v, and a new velocity vector obtained via the optical law of reflection. The billiard travels at a constant speed of v, and fixed points are when $||u|| = 1$. We prove: near the boundary of the phase space, the billiard map is a small perturbation of a time shift of the geodesic flow, and for a dense and open set of such surfaces Γ, there are trajectories of the billiard map for which $||u|| \longrightarrow 1$.

Sara Di Ruzza, University of Padua, Italy

The radioscience experiment with BepiColombo mission to Mercury

BepiColombo is a fundamental ESA/JAXA space mission, consisting of two spacecrafts orbiting around Mercury in order to achieve new important results regarding geophysics, geodesy and fundamental physics. The Mercury Orbiter Radioscience Experiment (MORE) is one of the on-board experiments, including three different but linked experiments: gravimetry, rotation and relativity. Using radio observables (range and range-rate) performed with very accurate tracking from ground stations, together with optical observations from the on-board high resolution camera (SIMBIO-SYS) and using accelerometer readings from the on-board accelerometer (ISA), MORE will be able to measure with unprecedented accuracy the global gravity field of Mercury, the rotation state (obliquity and libration motions) of the planet and will allow us to conduct modern tests on General Relativity based on relativistic light propagation near the Sun. This talk aims to present the state of the art of the experiment just before the upcoming launch in October 2018. The work is a collaboration with the Celestial Mechanics group of the University of Pisa.

Nathan Duignan, University of Sydney, Australia

Normal Forms, Blow-up and the Simultaneous Binary Collision

This talk explores the dynamics near the singularity at the simultaneous binary collision of the 4-body problem. It has been conjectured by Simo and Martinez that any attempt to remove the singularity via block regularisation will result in a regularised flow that is no more than $C^{8/3}$ differentiable with respect to initial conditions. Through a geometric desingularisation and study of the homological operator associated to the normal form of the singularity, we explore this curious loss of differentiability, providing a new proof of the conjecture for the collinear problem. The extension of our methods to the unresolved case of the planar problem is discussed.

Filippo Giuliani, Scuola Internazionale Superiore di Studi Avanzati, Italy

On the integrability of the dispersive Degasperis-Procesi equation

The Degasperis-Procesi equation ([4], [5])

$$u_t + c_0 u_x + \gamma u_{xxx} - 2\alpha u_{xxt} = \Big(c_2(u_x^2 + u u_{xx}) - 2\frac{c_3}{\alpha^2} u^2 \Big)_x \ , \quad c_0, \alpha, \gamma, c_2, c_3 \in \mathbb{R}$$

has been extensively studied by many authors, especially in its dispersionless form, since it presents interesting phenomena such as breaking waves and existence of peakon-like solutions ([2], [6], [9]). Degasperis-Holm-Hone [3] proved the integrability of this equation and they provided an iterative method to compute infinite conserved quantities. Since the Degasperis-Procesi equation is a quasi-linear equation the presence of dispersive terms depends on the chosen frame. In absence of dispersive terms there are no constants of motion even controlling the H1 -norm ([7]). We show that, in the dispersive case, we can construct infinitely many constants of motion which are analytic and control the Sobolev norms in a neighborhood of the origin. Moreover, thanks to the analysis of the algebraic structure of the quadratic parts of these conserved quantities we show that the (formal) Birkhoff normal form is action-preserving (integrable) at any order. This fact is used to find quasi-periodic small amplitude solutions following the scheme designed by BaldiBerti-Montalto [1] for the search of quasi-periodic solutions for quasi-linear PDEs (a joint work in preparation with R.Feola and M. Procesi). The study of the constants of motion for the dispersive Degasperis-Procesi equation is part of the work [8].

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Ioannis Gkolias, Politecnico di Milano, Italy

High Earth orbits' characterisation by Hamiltonian reduction on the ecliptic

Since the beginning of the space Era, the Hamiltonian formulation and the tools of the canonical perturbation theory, allowed to develop fast and accurate tools to predict the orbital evolution of Earth's artificial satellites. For close Earth satellites, analytical theories are usually based on an Earth-centred inertial frame that uses the equator as a reference plane. However, for high-altitude satellites, the lunisolar perturbations dominate the dynamics and they are more conveniently represented in reference frame which uses the ecliptic as a reference plane. The two representations are physically equivalent, but from a canonical perturbation theory point of view, there exist significant differences that we will try to exploit. Namely, in the ecliptic plane representation, after a first normalization over the satellite's and the Moon's orbital motions, it is possible to proceed with a further normalisation of the node of the satellite. The resulting Hamiltonian is of 1 degree-of-freedom and is no more time-dependent. This allows to proceed with a full characterisation of the phase-space, by locating the critical points of the reduced model and determining their stability. A further discussion on the bifurcation phenomena that take place in the parameter-space, allows to obtain useful information on the stability of the different orbital regions and to evaluate them in terms of preliminary orbit design for distant Earth satellite orbits.

Anne-Sophie Libert, Université de Namur, Belgium

Laplace-Lagrange hamiltonian expansion in extrasolar systems

Unlike the planets of the Solar System, the exoplanets present a wide variety of orbital eccentricities. Since the classical Laplace-Lagrange secular expansion uses the circular approximation as a reference, its use for describing the long-term evolution of extrasolar systems can be doubtful. We show here that possible extension of the Laplace-Lagrange theory reveals very efficient for three categories of planetary systems. For secular systems, an expansion to high order in eccentricities and inclinations accurately describes their long-term evolution, even for highly inclined systems. For systems that are near a mean-motion resonance, an extension of the LaplaceLagrange secular theory to order two in the masses is required. Finally, the long-term evolution of systems that are really close to or in a mean-motion resonance can be described by including appropriate resonant combinations of the fast angles into the Laplace-Lagrange expansion. As an application, we show how the refined analytical expansion can be used to identify ranges of mutual inclinations which ensure the long-term stability of non-resonant extrasolar systems. Joint work with M. Sansottera and M. Volpi.

Stefano Marò, Instituto de Ciencias Matemáticas, Spain

Aubry-Mather theory for conformally symplectic systems. (Joint work with Alfonso Sorrentino.)

We study the dynamics and the invariant sets for a class of dissipative systems, namely conformally symplectic systems. More precisley, we consider flows that do not preserve the symplectic structure but do alter it up to a constant scalar factor. The study of invariant Lagrangian submanifolds for these systems, in particular KAM tori, has been investigated by means of varied techniques in recent years. We will focus on what happens when these invariant Lagrangian submanifolds stop to exist. Inspired by the celebrated Aubry-Mather and Weak KAM theories for Hamiltonian systems, we prove the existence of interesting invariant sets, which, in analogy to the conservative case, will be called the Aubry and the Mather sets. We describe the structure and the dynamical significance of these sets, their attracting/repelling properties, as well as their noteworthy role in driving the asymptotic dynamics of the system.
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Mauricio Misquero Castro, Universidad de Granada, Spain

Resonance tongues in the Linear Sitnikov equation

In this research we deal with a Hill's equation, depending on two parameters $e \in [0,1)$ and $\Lambda > 0$, that has applications to some problems in Celestial Mechanics of the Sitnikov-type. Due to the nonlinearity of the eccentricity parameter e and the coexistence problem, the stability diagram in the (e,Λ)-plane presents unusual resonance tongues emerging from points $(0,(n/2)^2)$, $n = 1,2,...$ The tongues bounded by curves of eigenvalues corresponding to 2π-periodic solutions collapse into a single curve of coexistence (for which there exist two independent 2π-periodic eigenfunctions), whereas the remaining tongues have no pockets and are very thin. Unlike most of the literature related to resonance tongues and Sitnikov-type problems, the study of the tongues is made from a global point of view in the whole range of $e \in [0,1)$. Indeed, it is found an interesting behavior of the tongues: almost all of them concentrate in a small Λ-interval $[1,9/8]$ as $e \to 1^-$. We apply the stability diagram of our equation to determine the regions for which the equilibrium of a Sitnikov $(N+1)$-body problem is stable in the sense of Lyapunov and the regions having symmetric periodic solutions with a given number of zeros.

Federico Mogavero, Institut d'astrophysique de Paris, France

Rare Events in the Dynamics of the Inner Solar System Planets

According to Laskar & Gastineau (2009), there is a 1% probability of losing Mercury through catastrophic events in the inner solar system during the next 5 Gyr. Such a result was established via one thousand, very expensive direct simulations of planet dynamics. We are currently interested in assessing the probability of a very rapid (~ 100 Myr) destabilization of Mercury orbit. Such an event is very rare and a naive numerical approach would not be fruitful. I will explain the relevance of such an investigation, halfway between celestial mechanics and statistical physics, and illustrate our current approach based on Gauss' secular dynamics and rare-event-sampling algorithms.

Michel Orieux, Université Paris Dauphine, France

Singularities of minimum time control of mechanical systems

This poster will focus on recent developments on optimal time control of mechanical systems, with in mind application to the controlled Kepler and circular restricted three body-problems (CRTBP). We are interested in minimizing the final time for affine control systems,

$$\begin{cases} \dot{x}(t) = F_0(x(t)) + u_1(t)F_1(x(t)) + u_2(t)F_2(x(t)),\ t \in [0, t_f]\ , \quad u \in \mathcal{B} \\ x(0) = x_0\ , \quad x(t_f) = x_f \\ t_f \longrightarrow min \end{cases}$$

where the control u is contained in the euclidean ball B, the $F_i$are smooth vector fields, and the phase space $\mathcal{M}$ is a four dimensional manifold. (Most results are valid of 2 dimensional manifold with m controls). Pontrjagin Maximum Principle provides the following necessary condition: Optimal trajectories are projections on $\mathcal{M}$ of so-called extremals, that is of solutions of the Hamiltonian system defined on $T^*\mathcal{M}$ by

$$H^*(z) = H_0(z) + \sqrt{H_1^2(z) + H_2^2(z)},\ z = (x,p) \in\ T^*\mathcal{M}$$

with $H_i(z) = \langle p, F_i(z) \rangle\ , i = 0, 1, 2$. The associated control is

$$u = \frac{(H_1, H_2)}{\sqrt{H_1^2 + H_2^2}}\ .$$

The set $\Sigma = \{z \in T^*\mathcal{M} : H_1(z) = H_2(z) = 0\}$ defines a singular locus, and we are interested in the behaviour of the Hamiltonian flow in the neighborhood of Σ. We partition Σ into three subsets $\Sigma_+, \Sigma_-, \Sigma_0$, on which we study the flow. We make the following generic assumption:

$$\det(F_1(x), F_2(x), F_{01}(x), F_{02}(x)) \neq 0 \ \text{ for almost all } x \in \mathcal{M}\ .$$

This assumption is in particular valid for every second order controlled mechanical sys- tem of the form (V denotes a potential)

$$\ddot{q} + \nabla V(q) = u\ .$$

We use a blow up and give a normal form for the extremal system, which allows us to prove:

Theorem 1. *Assume that the previous generic assumption holds, then there is a unique extremal (with a switch) passing through each $\bar{z} \in \Sigma_-$; the extremal flow is locally well defined and there exist a stratification of the phase space such that the flow is smooth on each stratum. Furthermore, when crossing the strata, the flow admits log-type singularities (and thus belongs to the log-exp category).*

In a tubular neighborhood of Σ_+, the flow is smooth and no extremal crosses Σ_+ (thereis no switch, thus). In the last case (Σ_0), we prove the following:

Proposition 2. *For every point $\bar{z} \in \Sigma_0$, there exists an extremal having a switch at $\bar{z}$.*

This is a joint work with J.-B. Caillau (Nice), J. Féjoz (Paris) and R. Roussarie (Dijon).

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Victor Ortega, Universidad de Granada, Spain

Point-vortex stability under a periodic perturbation.

I will present a result about the stability of a periodic Hamiltonian system in the plane with a singularity. In a perfect fluid, a point-vortex is essentially a singularity of the vorticity, and can be modeled by the Hamiltonian $\Psi_0(x,y) = \frac{1}{2}\ln(x^2+y^2)$ being x and y the usual rectangular coordinates in the plane. The associated system is integrable, with the particles rotating around the vortex in circular paths and the origin is stable. If we introduce a periodic perturbation $p(t,x,y)$, the corresponding Hamiltonian system is

$$\begin{cases} \dot{x} = \frac{y}{x^2+y^2} + \partial_y p(t,x,y) \\ \\ \dot{y} = -\frac{x}{x^2+y^2} - \partial_x p(t,x,y) \end{cases} \qquad (x,y) \in \mathcal{U} \setminus \{0\}$$

where $\mathcal{U}$ is a neighborhood of the origin. This system models ideally the passive advection (transport) of particles in a fluid subjected to the action of a steady vortex placed at the origin and a time-dependent background flow.
We will see which hypothesis must be imposed on the pertubation $p(t,x,y)$ to preserve the stability of the origin. In this context, the application of Moser's twist theorem allows us to find a family of invariant curves by the Poincaré map of our system. This is important since these curves surround the origin

and act as barriers to the solutions; therefore, the stability of the origin will be guaranteed.

Reference:

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<u>**Rocio Paez**</u>, University of Padua, Italy

Modeling chaotic diffusion along resonances: fastest drift orbits

Several types of dynamical systems appearing in Celestial Mechanics can be represented by simple models of chaotic diffusion along resonances. The clarification of the diffusion mechanisms becomes then an important question. In the present work, we will focus on the use of a technique based on normal form computations, that allows to unveil the fastest diffusing chaotic orbits. In particular, we implement a normalization process that completely erases the deformation effects over the orbits' evolution in action space. This, in turn, makes possible to visualize the chaotic diffusion in the separatrix domain of the resonant chaotic layers. Furthermore, we demonstrate that only a few terms in the remainder of the optimal normal form drive the chaotic jumps in action space. We obtain precise quantitative estimates of the diffusion rate, by implementing a Melnikov-type analysis of the dynamics induced by the above remainder terms. Finally, we discuss some practical applications of the computation of such fast drift orbits in real problems of Celestial Mechanics.

<u>**Tiziano Penati**</u>, University of Milano, Italy

On the continuation of degenerate periodic orbits via normal form with some applications.

We reconsider the classical problem of the continuation of degenerate periodic orbits in nearly integrable Hamiltonian systems. In particular we focus on periodic orbits that arise from the breaking of a completely resonant maximal torus. We propose a suitable normal form construction that allows to identify and approximate the periodic orbits which survive to the breaking of the resonant torus. Our algorithm allows to treat the continuation of approximate orbits which are at leading order degenerate, hence not covered by the classical averaging methods. We discuss possible extensions to low dimensional tori and applications to existence and stability of localized periodic orbits in chains of weakly coupled oscillators.

<u>**Alessandro Portaluri**</u>, University of Torino, Italy

Index and stability of closed (semi-)Riemannian geodesics

A celebrated result due to Poincaré asserts that a closed minimizing geodesic on an oriented surface is linearly unstable when considered an orbit of the co-geodesic flow. In this talk, starting from this classical theorem, we discuss some recently new results on the instability and hyperbolicity of closed (maybe not minimizing) geodesics of any causal character on higher dimensional (even not oriented) semi-Riemannian manifolds. Dropping the non-positivity assumption of the metric tensor is a quite challenging task since the Morse index is truly infinite.

Alexandre Pousse, University of Padua, Italy

On the stability of the Saturn co-orbital moons Janus and Epimetheus in the three-body problem (Joint work with P. Robutel and L. Niederman)

In the framework of the planetary three-body problem (two bodies orbiting a more massive one), the co-orbital motion is associated with trajectories in 1:1 mean-motion resonance. In other words, the planets share the same orbital period. This problem possesses a very rich dynamics which is related to the five famous "Lagrange" configurations. This resonance has been extensively studied since the discovery of Jupiter's "Trojan" asteroids, whose trajectories librate around one of the L4 and L5 equilibria with respect to the Sun and the planet. Other co-orbital objects have been discovered in the system of Saturn's satellites such as the pair Janus-Epimetheus which exhibits a peculiar dynamics associated with horseshoe-shaped trajectories.
As they orbit Saturn (in about 17 hours) on quasi-coplanar and quasi-circular trajectories whose radii are only 50 km apart (less than their respective diameters), their mean orbital frequency is slightly different (the inner body being a little faster than the outer one). Thus, the bodies are getting closer every four years and their mutual gravitational influence leads to a swapping of the orbits. The outer moon becoming the inner one and vice-versa, this behavior generates the horseshoe trajectories depicted in an adequate rotating frame. This surprising dynamic of the Janus-Epimetheus co-orbital pair was confirmed by Voyager 1 flyby in 1981. Since then, several analytical theories have been developed to describe their long-term dynamics and, more generally, of horseshoe motions in the three-body problem. However, in spite of these works as well as the indications provided by some numerical investigations, so far no rigorous long time stability results have been obtained even in the restricted three-body problem. Following the idea of Arnold (1963), our goal was to provide a rigorous proof of the existence of invariant tori associated with the Janus and Epimetheus horseshoe motion in the planar three-body problem using KAM theory. To this aim, we had to define a suitable integrable approximation of the system that can be suggested by the astronomical observations. However, our context was much more tricky: unlike Arnold's situation that relies on non-resonant Kepler orbits, we are strictly in 1:1-resonance which prevent to use the secular perturbation in order to get a non-degeneracy. Thus, during the talk, I will present our strategy in order to build an adapted approximation of the system. A drawback of this method is that it is very tricky to check Kolmogorov non-degeneracy condition as in Arnold's article. Hence, I will also explain how we overcame this difficulty by implementing a more recent KAM theorem developed by H. Russman and M.

Herman.

Marco Sansottera, University of Milano, Italy

A reverse KAM *method to estimate unknown mutual inclinations in exoplanetary systems*

The orbital evolution of the major planets of the Solar system is usually studied in the framework of the Lagrange-Laplace secular theory (i.e, averaging the system over the fast revolution angles). In this approximation the evolution of the eccentricities and inclinations is quasi-periodic: the corresponding orbits are stable [3], lying on KAM tori or, at least, being close to them for times exceeding the age of the Universe [1, 2]. The applicability of KAM theorem for a secular model of the Sun-Jupiter-Saturn system has been done in [4], where the behavior of the mutual inclination is obtained from the so-called Angular Momentum Deficit (precisely the D2 parameter introduced in [5]). Coming to extrasolar systems, the inclinations of exoplanets detected via radial velocity method are essentially unknown. We aim to provide estimations of the ranges of mutual inclinations that are compatible with the long-term stability of the system. Focusing on the skeleton of an extrasolar system, i.e., considering only the two most massive planets, we tackle the converse problem: assuming that the system is stable, we aim to bound the usually unknown mutual inclination to a suitable range of values. Precisely, we investigate the range of values of D2 for which KAM stability applies to the secular dynamics. Such a procedure seems to be successful in providing limits on the (unknown) inclination for pairs of exoplanets having very moderate eccentricities, i.e., in situations similar to the Sun-Jupiter-Saturn system. We show our first applications to the following extrasolar systems: HD141399, HD143761 and HD40307. Joint work with U. Locatelli and M. Volpi [6].

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Arturo Vieiro, Universitat de Barcelona, Spain

Splitting of the separatrices after a Hamiltonian-Hopf bifurcation under periodic forcing

We consider a 2-dof autonomous Hamiltonian system obtained as a truncation of the Hamiltonian-Hopf normal form. For concreteness assume that the eigenvalues at the fixed point are $\pm\nu \pm i$ where $\nu = 0$ at the bifurcation and $\nu > 0$ when the complex saddle has been created. We add to this system a very simple non-autonomous periodic perturbation. We study the splitting of the invariant 2-dimensional stable/unstable manifolds after the perturbation, mainly using first order Poincaré - Melnikov approach. We analyse the different changes of dominant harmonic(s) in the splitting functions when the small parameter ν changes and how these changes depend on the quotients of the continuous fraction expansion of the periodic forcing frequency. The methodology used is general enough to systematically deal with different frequencies including quadratic irrationals, frequencies with a bounded sequence of quotients of the continuous fraction expansion and frequencies with unbounded sequence of quotients. The results are compared with a direct numerical computation of the invariant manifolds and we obtain an excellent agreement between the data obtained with both methods. However with our methodology we can reach much smaller values of ν. This is a joint work with E. Fontich and C. Simó.

Guowei Yu, University of Turin, Italy

An Index Theory for Zero Energy Solutions of the Planar Anisotropic Kepler Problem

In the variational study of singular Lagrange systems, the zero energy solutions play an important role. In this talk, we will explain a simple way of computing the Morse indices of these solutions for the planar anisotropic Kepler problem. In particular this reveals an interesting connection between the Morse indices and the oscillating behaviors of these solutions. Our results can be applied to the Kepler problem and isosceles three body problem as well. This is a joint work with Xijun Hu.