H2020 in Hamiltonian Dynamics

Abstracts

Alberto Abbondandolo [Wed 9:00–9:50]

Symplectic capacities and normal forms

In 2000, Claude Viterbo proposed an intriguing conjecture concerning the symplectic capacities of convex bodies in the standard 2n-dimensional phase space. I will discuss some ideas in the proof of a perturbative version of this conjecture. This proof, which is part of a joint work with Gabriele Benedetti, involves a new normal form for Reeb flows which are close to periodic flows.

Dario Bambusi [Thu 9:55–10:45]

A Quantum Nekhoroshev Theorem: growth of Sobolev norms in quasi integrable quantum systems

I will present an abstract result giving a $\langle t\rangle^{\epsilon}$ upper bound on the growth of the Sobolev norms of a time dependent Schrödinger equation of the form

$$\mathrm{i}\dot{\psi} = H_0\psi + V(t)\psi \ .$$

H_0 is assumed to be the Hamiltonian of a steep, globally integrable quantum system and to be a pseudodifferential operator of order $\mathtt{d} > 1$; essentially it is the quantization of a classical steep integrable system. $V(t)$ is a time dependent family of pseudodifferential operators, unbounded, but of order $\mathtt{b} < \mathtt{d}$.

I will discuss our definition of globally integrable quantum system and show that quantum anharmonic oscillators in dimension 2, and the Laplacian on some manifolds with integrable geodesic flow fulfill this definition.

Then I will give the statement of the result and present some ideas of the proof which is a quantum version of the proof of the classical Nekhoroshev theorem (both analytic and geometric parts). Joint work with Beatrice Langella.

Massimiliano Berti [Thu 9:00–9:50]

Instability of Stokes waves

A classical subject in fluid mechanics regards the spectral instability of traveling periodic water waves, i.e traveling periodic solutions. Benjamin, Feir, Whitam and Zhakarov predicted, through experiments and formal arguments, that Stokes waves in sufficiently deep water are unstable. More precisely, they found unstable eigenvalues near the origin of the complex plane, corresponding to small Floquet exponents μ or equivalently to long-wave perturbations. The first rigorous mathematical results have been given by Bridges-Mielke ('95) in finite depth and by Nguyen-Strauss ('20) in infinite depth. On the other hand, it has been found numerically that when the Floquet number μ varies, two eigenvalues trace an entire figure-eight. I will present a novel approach to prove this conjecture fully describing the unstable spectrum. It exploits the Hamiltonian and reversible properties of the water waves, a symplectic version of Kato's theory of similarity transformations, and a block diagonalization idea, inspired by KAM theory. This is joint work with A. Maspero and P. Ventura.

Abed Bounemoura [Mon 12:10–13:00]

Small divisors problems in non quasi-analytic classes

For the linearization problems of circle diffeomorphisms and germs, we will explain optimal results in Gevrey classes. Conjecturally, this should be true for any small divisors problems in any non quasi-analytic class.

Alessandra Celletti [Mon 9:55–10:45]

Perturbative methods in Earth's space debris dynamics

Space debris represent a real threat for operative satellites and space missions; understanding their dynamics is of paramount importance. In particular, it is crucial to understand the location of the equilibria, the existence of regular or chaotic regions, and especially their stable and unstable behavior as some control parameters are varied. These studies might allow to develop mitigation, maintenance and control strategies based on mathematical investigations.

To analyze the dynamics of space debris, we implement the following perturbative methods:

(i) normal forms to get the long-term behaviour of the orbits,

(ii) Nekhoroshev's theorem to obtain exponential stability times,

(iii) a multi-scale normalization procedure to compute the proper elements, which are quasi-invariants of the dynamics. The proper elements allow us to identify groups of fragments associated

to the same break-up event and to back-trace the fragments to a parent body. These results are corroborated by statistical data analysis.
This talk refers to different works made in collaboration with: I. De Blasi, C. Efthymiopoulos, C. Gales, C. Lhotka, G. Pucacco, T. Vartolomei.

Andrew M. Clarke [Thu 17:00–17:30]
Arnold Diffusion in the Secular Four-Body Problem
We consider the four–body problem, with arbitrary fixed masses, in the regime where the bodies revolve far apart from each other in space. In particular, we assume that the semi–major axes of the Keplerian ellipses are of different orders, and that the mutual inclination between the first two planets is non-negligible. Under these assumptions, we prove that there are orbits of the four-body problem where the inclination and eccentricity of the second planet follows any predetermined itinerary. The proof is analytical, and combines some classical ideas from celestial mechanics (Legendre polynomials, averaging) as well as from the theory of Arnold diffusion (normally hyperbolic invariant manifolds, scattering maps) and some newer ideas such as Deprit coordinates and a tailor-made shadowing argument based on the method of correctly aligned windows. Moreover, the latter method provides us with (non-optimal) time estimates. In addition, the proof applies to the planetary regime where there is one massive body (the sun) and three small masses (the planets). The results are based on joint work with Jacques Féjoz and Marcel Guardia.

Amadeu Delshams [Mon 11:15–12:05]
Global instability based on shadowing along non-transverse heteroclinic chains
We discuss a geometric method called *dropping dimensions mechanism*, based on shadowing a chain of N non-transverse heteroclinic connections by a disk that releases dimensions at each transition (change of connection). After N transitions, what is left of the disk is a single point and we cannot go any further. We illustrate this new mechanism in a generalization of the toy model systems introduced by Colliander et al.; Guardia and Kaloshin and Guardia, Hauss and Procesi; for the study of the transfer of energy to high frequencies in defocusing nonlinear Schrodinger equations. This is a joint work with Piotr Zgliczynski (Jagiellonian U., Krakow).

Sara Di Ruzza [Mon 14:30–15:20]

Chaos in the simply averaged three-body problem

In this talk we present two numerical studies of the three–body problem in the planar case. The first one concerns the motion of two bodies having equal mass (binary asteroids) and perturbed by a third one which is much heavier (a planet) and far away. The second one deals with 3 bodies with comparable masses focusing in the region of the unperturbed separatrix, which is complicated by a collision singularity; we study the variation of a certain function, which (for a reason that we shall describe) we call "Euler Integral" around an orbit which spends much time closely to the saddle point of a manifold where the Euler integral is constant. In both cases we reduce the Hamiltonian from 3 to 2 degrees of freedom and discuss in detail the 3–dimensional phase space through the use of first return mappings. Moreover, we show the existence of chaos, via the machinery of symbolic dynamics, developed by M. Gidea and P. Zgliczynski. This is joint work with Jérôme Daquin and Gabriella Pinzari.

Jacques Féjoz [Tue 9:00–9:50]

Scattering of N particles in a long range potential

In this work with Andreas Knauf and Richard Montgomery, we consider the classical scattering of N particles in a long range potential ($O(r^{-a})$) as r tends to infinity, with $a > 0$. We define a "free region", subset of initial conditions leading to well defined and separated final states. In this region, the "final state" map is smooth and the dynamics is integrable, conjugate to the free dynamics.

Vassili Gelfreich [Wed 11:15–12:05]

What is this Arnold Diffusion anyway?

Arnold Diffusion has been at the centre of research efforts of mathematicians for several decades. The problem is related to description of dynamics outside a set of invariant tori in a nearly integrable Hamiltonian system with three or more degrees of freedom. High dimensionality of such systems poses a serious challenge for achieving a comprehensive description of dynamics, and many successful methods developed for studying Arnold Diffusion are abstract in their nature.

In this talk I will show that an adaptation of the ideas from the classical Lochak-Neishtadt's proof of the Nekhoroshev theorem in combination with the discrete averaging method can be used to provide a transparent picture of dynamics in four-dimensional symplectic maps.

We will see that the phase space of a nearly integrable map has a covering such that inside every domain an iterate of the map is approximated with exponential accuracy by an explicitly computable

Hamiltonian system. This system has two degrees of freedom and typically is not integrable. We will see that it may have large chaotic zones on energy levels located near a nucleus, but the system is close to integrability near the boundary of its domain.

We use the technics of interpolating vector fields to obtain high-resolution illustrations of four-dimensional dynamics.The usual features of dynamics in two-degrees of freedom (invariant tori, chains of resonant islands, chaotic zones) have easily recognisable reflections in the dynamics of the four-dimensional symplectic map.

Moreover, the corresponding Hamiltonian function represents an explicitly computable slow variable. We use this variable to explore diffusive properties of the Arnold diffusion. In particular, we see that on certain timescales the Arnold Diffusion behaves numerically like a diffusive process, but the diffusion coefficient is highly sensitive to the position of initial conditionsinside the phase space.

The talk is based on joint works (some in progress) with Arturo Vieiro.

Guido Gentile [Fri 9:00–9:50]

Almost-periodic solutions for the one-dimensional NLS equation

We consider the NLS equation with a convolution potential and prove the existence of almost-periodic solutions assuming that the Fourier multipliers decay polynomially. Joint work with L. Corsi and M. Procesi.

Marian Gidea [Fri 9:55–10:45]

Arnold diffusion in dissipative systems

We consider a mechanical system consisting of a rotator and a pendulum coupled via a small, time-periodic Hamiltonian perturbation, and subject to a damping perturbation. The resulting system has energy dissipation. We provide explicit conditions on the damping parameter, so that the resulting dissipative system exhibits Arnold diffusion. More precisely, we show that there are diffusing orbits along which the energy of the rotator grows by an amount independent of the size of the coupling parameter, for all sufficiently small values of the coupling parameter. The fact that Arnold diffusion may play a role in systems with small dissipation was conjectured by Chirikov.

Joint work with: S.W. Akingbade and T. M-Seara.

Marcel Guardia [Tue 9:55–10:45]

Oscillatory motions and symbolic dynamics in the three body problem

Consider the three body problem with positive masses m_0, m_1 and m_2. In 1922 Chazy classified the possible final motions the three bodies may possess, that is the behaviors the bodies may have when time tends to infinity. One of them are what is known as oscillatory motions, that is, solutions of the three body problem such that the liminf (as time tends to infinity) of the relative positions between bodies is finite whereas the limsup is infinite. That is, solutions for which the bodies keep oscillating between an increasingly large separation and getting closer together. The first result of existence of oscillatory motions was provided by Sitnikov for a Restricted Three Body Problem, called nowadays Sitnkikov model. His result has been extended to several Celestial Mechanics models, but always with rather strong assumptions on the values of the masses. In this talk I will explain how to construct oscillatory motions for the three body problem for any values m_0, m_1 and m_2 (except for the case of three equal masses). The proof relies on the construction of hyperbolic invariant sets whose dynamics is conjugated to that of the shift of infinite symbols (i.e. symbolic dynamics). That is, we construct invariant sets for the three body problem with chaotic dynamics, which moreover contain oscillatory motions. This is a joint work with Pau Martin, Jaime Paradela and Tere M. Seara.

Massimiliano Guzzo [Tue 11:15–12:05]

Measures of slow diffusion in Hamiltonian Systems

The long-term diffusion in Hamiltonian systems with more than two degrees of freedom has been represented, since Arnold's example, as a slow diffusion within the so called Arnold web, an intricate web of the phase-space formed by chaotic trajectories. With modern computers it is possible on the one hand to perform numerical integrations which reveal the web of resonances as well as a slow diffusion occurring through it, on the other hand to compute resonant normal forms with an explicit representation of the remainders. A recent method developed by Guzzo, Efthymiopoulos and Paez, based on a stationary-phase approach to the analysis of the normal forms, provides quantitive predictions of the speed of the slow diffusion along the single resonances in agreement with the numerically computed ones, for a three-degrees of freedom Hamiltonian system.

Raphaël Krikorian [Mon 15:25–16:15]

Perturbations of conservative real analytic diffeomorphisms of the disk or the cylinder: KAM theory and straightening of almost complex structures.

Let TWIP be the set of twist (i.e. deviating the vertical) diffeomorphism of the disk or the cylinder which have the intersection property (they are "conservative"). I will show that any real analytic TWIP diffeomorphism having a C^3-invariant KAM curve (i.e. an essential circle on which the dynamics is conjugated to a Diophantine translation) is accumulated in the analytic topology by real analytic TWIPs each admitting a (small) invariant open annulus (one can "thicken" by perturbation the invariant circle). If time permits I will also describe how one can construct "non trivial" real analytic TWIPs with zero topological entropy. I will discuss what kind of results one can expect in the symplectic case (area preservation).

The proofs of these results are based on KAM theory and an argument of straightening of integrable almost complex structures.

Sergei Kuksin [Thu 12:10–13:00]

Random perturbations of integrable systems

I will discuss behaviour of finite-dimensional integrable systems, linear and non-linear, under stochastic perturbations of size epsilon, and will present three theorems. The first treats solutions on time-intervals of order 1/epsilon, the second discuss their asymptotic behaviour as time goes to infinity, and the last controls the behaviour of solutions uniformly in time.

Jessica E. Massetti [Tue 12:10–13:00]

On the persistence of periodic tori for symplectic twist maps

Invariant tori that are foliated by periodic points are at the core of the fragility of integrable systems since they are somehow extremely easy to break, in counterposition to the generic robustness of the quasi-periodic ones considered by KAM theory. On the other hand, the investigation of rigidity of integrable twist maps, i.e. to understand to which extent it is possible to deform a map in a non-trivial way preserving some (or all) of its features, is related to important questions and conjectures in dynamics. In this talk I shall discuss the persistence of Lagrangian periodic tori for symplectic twist maps of the 2d-dimensional annulus and a rigidity property of completely integrable ones. This is based on a joint work with Marie-Claude Arnaud and Alfonso Sorrentino.

Federico Mogavero [Tue 17:00–17:30]

The origin of chaos in the Solar System through computer algebra

The discovery of the chaotic motion of the planets in the Solar System dates back more than 30 years. Still, no analytical theory has satisfactorily addressed the origin of chaos so far. Implementing canonical perturbation theory in the computer algebra system TRIP, we systematically retrieve the secular resonances at work along the orbital solution of a forced long-term dynamics of the inner planets. We compare the time statistic of their half-widths to the ensemble distribution of the maximum Lyapunov exponent and establish dynamical sources of chaos in an unbiased way. New resonances are predicted by the theory and checked against direct integrations of the Solar System. The image of an entangled dynamics of the inner planets emerges.

Laurent Niederman [Thu 15:25–16:15]

Semi-algebraic geometry and generic long-time stability of nearly-integrable Hamiltonian systems

A well-known theorem due to Nekhoroshev in 1977 shows that if we consider a sufficiently regular integrable Hamiltonian which satisfy a transversality property - known as "steepness" - then, adding a smooth small enough Hamiltonian perturbation, the solutions of the perturbed system exist and are stable over a very long time. Nekhoroshev also showed in [1] that the steepness is an open property, which is generic both in measure and topological sense in the space of jets (Taylor polynomials) of sufficiently smooth functions. Surprisingly, this latter result remained almost unstudied up to now, while the other parts of Nekhoroshev's theory received a lot of attention over the decades. Moreover, the definition of steepness is not constructive and no general criteria to ensure that a given function is steep existed up to now. This is a major problem in view of the applications.

We revisit Nekhoroshev's article [1] about the genericity of steepness and connect this proof with much more recent seminal articles of Roytwarf-Yomdin [2] and Yomdin [3] in semi-algebraic geometry. This allows to give a good setting for the considered problem and to clarify the reasoning.

In this framework, thanks to a deep understanding of the genericity of steepness, Santiago Barbieri has developed an explicit algebraic criterion in the space of jets of any order and any number of variables which ensure that a given function is steep.

Joint work with Santiago Barbieri.

References:

[1] N. N. Nekhoroshev: Stable lower estimates for smooth mappings and for gradients of smooth function, Mathematics of the USSR-Sbornik, 1973, vol. 90 (132), no. 3, pp.432-478.

[2] N. Roytwarf and Y. Yomdin: Bernstein Classes, Annales de l'Institut Fourier, 47(3):825-858, 1998.

[3] Y. Yomdin: Analytic reparametrization of semi-algebraic sets, J. Complexity, 24:54-76, 2008.

Michela Procesi [Thu 11:15–12:05]

Reducibility and nonlinear stability for a NLS equation on T^2

The problem of existence and linear stability of invariant tori for the NLS equation on T^2 has been studied by various authors. Of course the next step is to go to nonlinear stability. I shall discuss this question on the simplified model of a non-resonant quasi-periodic in time NLS equation. Joint work with E.Haus, B. Langella, A. Maspero.

Benedetto Scoppola [Wed 9:55–10:45]

Lonely planets and lightweight asteroids: A statistical mechanics model for the planetary problem

In a statistical mechanics context we propose a notion of stability for systems of particles interacting via newtonian potential and orbiting a much bigger object. The usual conditions requested in statistical mechanics in order to perform the thermodynamic limit fails in this case, but one can assume that the relevant parameter is the number of particles N that guarantees our notion of stability. Both for belts of very light objects and for heavier planets satisfying a kind of Titius-Bode Law we find that our notion of stability fits with the data observed in the Solar System. Joint work with Gabriella Pinzari and Alessio Troiani.

Tere M-Seara [Fri 11:15–12:05]

Some results about the non-existence of small breathers in Klein-Gordon equations

Breathers are solutions of evolutionary PDEs, which are periodic in time and spatially localized.
They are known to exist for the sine-Gordon equation but are believed to be rare in other Klein-Gordon equations.

When the spatial dimension is equal to one, working in the spatial dynamics framework (which consists in exchanging the roles of time and space variables), breathers can be seen as homoclinic solutions to steady states in an infinite dimensional phase space consisting on periodic in time solutions.

These homoclinic orbits arise, as usual, from the intersections of the stable and unstable manifolds of the steady states.

In this talk, taking the temporal frequency as a parameter, we shall study small breathers of the non-linear Klein-Gordon equation generated in an unfolding bifurcation as a pair of eigenvalues collide at the origin.

Due to the presence of the oscillatory modes, generally the finite dimensional stable and unstable manifolds do not intersect in the infinite dimensional phase space, but with an exponentially small splitting (relative to the amplitude of the breather) in this singular perturbation problem of multiple time scales.

When the steady solution has weakly hyperbolic one dimensional stable and unstable manifolds we will prove an asymptotic formula for their distance.

This formula allows to say that for a wide set of Klein-Gordon equations breathers do not exist.

Susanna Terracini [Wed 12:10–13:00]

TBA

Dmitri Turaev [Tue 14:30–15:20]

On adiabatic control of Schroedinger equation

We show that slow cyclic change of parameters of a quantum system can lead to a non-trivial permutation of quantum states if an additional quantum number is created and destroyed during the cycle of the perturbation. The repetition of the process can result in the exponential energy growth. Variations of the rate of the parameter change provide a new way of controlling the quantum state.

Piotr Zgliczynski [Thu 14:30–15:20]

Oscillatory Motions and Parabolic Manifolds at Infinity in the Planar Circular Restricted Three Body Problem

Consider the Restricted Planar Circular 3 Body Problem with both realistic mass ratio and Jacobi constant for the Sun-Jupiter pair. We prove the existence of all possible combinations of past and future final motions. In particular, we obtain the existence of oscillatory motions. All the constructed trajectories cross the orbit of Jupiter but avoid close encounters with it. The proof relies on the method of covering relations (correctly aligned windows) and is computer assisted. This is a joint work with M. Capinski, M. Guardia, P. Martin and T. Seara.

Ke Zhang [Tue 15:25–16:15]

Uniform Lyapunov exponents for Hamilton-Jacobi equations at the vanishing viscosity limit

It is well known that the viscous Hamilton-Jacobi equation on a compact domain converges exponentially fast to a stationary solution. (For example, Sinai proved this for a random potential on the torus in the late 80s). However, the a priori exponent decreases to 0 as the vicosity decreases to 0. On the other hand, in the zero viscosity case, the solution converges exponentially fast if the associated Lagrangian system admits a unique, hyperbolic minimizing orbit. We will show that for a generic "kicked" potential, the uniform convergence rate carry over to systems with small viscosity. This is a joint work with Konstantin Khanin and Lei Zhang.