Bogoliubov dispersion relation and the low Mach number limit for quantum fluids

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We consider the low Mach number regime for mathematical models for quantum fluids, such as the Quantum Hydrodynamics system and the Quantum Navier-Stokes equations. The study of the low Mach number limit is based on the analysis of the acoustic waves satisfying a system of linearised equations. While the dispersion of the acoustic waves in compressible fluids is linear, the dispersion for the considered quantum fluid models is governed by the Bogoliubov dispersion law. To that end, we develop refined Strichartz estimates for the linearised system that take in account the augmented dispersion relation. The motivation for the study of the linearised system is two-fold. Firstly, it allows us to carry out the low Mach number limit of finite energy weak solutions without further regularity assumptions for the aforementioned models. For the former one, we obtain convergence to a solution of the incompressible Navier-Stokes equations, while for the latter one we obtain convergence to incompressible Euler equations. Secondly, in the framework of the QHD system, the low Mach number scaling is suitable to study the behavior of quantized vortices. If the initial data contains a finite number vortices and is almost energy minimising, then in the limit the vortex motion law is given by the classical Kirchhoff law. This result has been known in the framework of Gross-Pitaevskii equations, due to J. Colliander, R. Jerrard et. al. and F. Lin, X. Xin and is recasted here in the framework of the Quantum Hydrodynamics.

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