PLENARY SPEAKERS

The role of vanishing pressure and viscosity in Navier-Stokes-Poisson models

Yann Brenier
CNRS, Université de Nice, France

One dimensional dynamics of pressure-less gases with sticky collisions is now fairly well understood and simple formulations relying on convex analysis have been recently obtained (Natile-Savare and the author, for instance). In addition, they can be derived from Navier-Stokes equations with vanishing pressure and viscosity. In presence of self-consistent (Newtonian or Coulombian) forces, it is not clear whether or not particles can unstick after collisions. In the Coulombian case, particles usually unstick after a finite time, as shown by numerical simulations, but, as explained in the talk, this mechanism may depend on the balance between pressure and viscosity as they both vanish.

Multidimensional discontinuities and free boundary problems in hyperbolic conservation laws

Gui-Qiang Chen
University of Oxford, UK

In this talk we will present several free boundary problems for the stability of multidimensional discontinuities and the existence of fundamental wave patterns in hyperbolic conservation laws. The discontinuities include shock waves, vortex sheets, and entropy waves. Some recent developments will be reviewed and discussed. Further trends, perspectives, and open problems in this direction will be also addressed.

Hidrodynamic models with singular constitutive laws

Peter Constantin
University of Chicago, USA

I'll talk about the non-viscous SQG equation and its generalizations. After a brief motivation and description of the state of the art, I'll show two results: one of well-posedness of active scalar equations with very singular constitutive laws and one about global regularity for an equation slightly more singular than 2D Euler.

Weakly nonlinear surface waves for hyperbolic initial boundary value problems

Jean-François Coulombel
CNRS, Université de Lille 1, France

This talk will present results on the formal derivation and rigorous justification of amplitude equations that govern weakly nonlinear surface waves for nonlinear hyperbolic initial boundary value problems. Following the first contributions on nonlinear elasticity, we shall show that weakly nonlinear surface waves are governed by nonlocal generalizations of the Burgers equation. We shall review the well-posedness theory for such amplitude equations in Sobolev spaces. When the initial boundary value problem has a variational origin, as in nonlinear elasticity, we shall show that the amplitude equation has an underlying Hamiltonian structure. This generalizes and justifies earlier observations by Hunter and coworkers. The talk is based on collaborations with Sylvie Benzoni-Gavage and Nikolay Tzvetkov.

Steady self-similar flow

Volker Elling
University of Michigan, USA

We consider solutions of the 2-d compressible Euler equations that are steady and self-similar. They arise naturally at interaction points in genuinely multi-dimensional flow. We characterize the possible solutions in the class of flows L-infinity-close to a constant supersonic background. We show they are necessarily BV. As a special case we prove that solutions of 1-d Riemann problems are unique in the class of L-infinity-small self-similar functions. (Joint work with Joseph Roberts)

Recovering singularities in quantum scattering theory

Daniel Faraco
Universidad Autónoma de Madrid, Spain

A recurrent theme in the scattering theory of the Schrödinger Hamiltonian $H = -\Delta + q$ is to determine how much information of the potential q can be recovered from the so called far field patterns or scattering amplitudes,

$$u_\infty(k,\theta,\theta') = \int_{\mathbf{R}^n} e^{-ik\theta'\cdot y} q(y) u(k,\theta,y)\, dy\,. \tag{1}$$

Here $u(k,\theta,\theta')$ is the solution to the problem

$$\begin{cases} (H-k^2)u = 0 \\ u = u_i + u_s \\ \lim_{|x|\to\infty} \left(\frac{\partial u_s}{\partial r} - iku_s\right)(y) = o(|y|^{-(n-1)/2}) \end{cases}$$

The solutions $u(k,\theta,y)$ are the responses of the electrical potential to an incoming free plane wave $u_i(y) = e^{ik\theta\cdot y}$ with wave number k, incoming angle $\theta \in \mathbb{S}^{n-1}$ and energy $= k^2$.
There are a number of theories depending on for which set of pairs $(\theta,\theta') \subset \mathbb{S}^{n-1}\times\mathbb{S}^{n-1}$ the scattering amplitude (1), is available. I will report on recent progress on the full data case (we know $u_\infty(k,\theta,\theta')$ for every (θ,θ')) and on the backscattering case (we know $u_\infty(k,\theta,\theta')$ for $\theta=\theta'$). The main tool in both cases is a refined analysis of bilinear Fourier multipliers. This is a joint work with J.A Barceló, A. Ruiz and A. Vargas.

The mean-field limit for a regularized Vlasov-Maxwell dynamics

François Golse
Ecole Polytechnique, Paris, France

In 1977, Braun and Hepp established the mean-field limit for a system of particles leading to a regularized variant of the Vlasov-Poisson system, where the Coulomb potential is replaced by a smooth kernel. Their result was later strengthened by Dobrushin in 1979, who estimated the rate of convergence in that limit. An analogue of their results for the relativistic Vlasov-Maxwell system will be presented.

Selfimproving convexity and Orstein's L-one noninequalities

Bernd Kirchheim
University of Oxford, England

Vectorial problems in the Calculus of Variations are naturally related to generalized notions of convexity - which for instance characterize weak lower semicontinuity locally or globally. In the past, mainly the differences between these notions were investigated. Here we present a result which shows that for one-homogeneus functions all these possible convexity notions agree in the best possible way.

This result, obtained jointly with Jan Kristensen, also allows to put Ornstein's famous L-one non-inequality as well as similar constructions by Conti, Faraco, Maggi and Mueller into a unifying frame and to generalize them.

Multi-dimensional gas flows

Tai-Ping Liu
Academia Sinica, Taiwan

We will discuss the question of existence and uniqueness of solutions for gas flows around a ramp. Historical perspectives and modern interpretations are emphasized. Key ideas and future prospects are illustrated through concrete problems.

Nonlinear Schrödinger equations and quantum fluids with critical nonlinear and Hartree potentials

Pierangelo Marcati
Università dell'Aquila, Italy

I shall present some recent development in the theory of Nolinear Schrödinger equation (in collaboration with P.Antonelli and G.Staffilani) regarding scattering and asymptotic completness in presence of nonlinear and Hartree potentials. I will undeline math difficulties due to the Hartree potentials at certain critical scale and the interaction with the study of quantum fluids.

Hyperbolic degeneracy in the calculus of variations of surfaces with finite total curvature

Tristan Rivière
ETH, Zurich, Switzerland

Surfaces which are critical for the L^2 norm of their second fundamental form have been introduced in the early XXth century in an attempt to merge minimal surface theory and conformal invariance. They were called at the time conformal-minimal surfaces and their attached lagrangian arises nowadays in various areas of science such as elasticity, general relativity, cell biology, optics...etc.
The calculus of variation of the L^2-norm of the second fundamental form of surfaces has been intensively studied in the recent years. This is a challenging 4th order problem in geometric analysis where in particular the interplay

between phenomena such as gauge invariance, concentration compactness, compensation compactness has to be understood.
One of the difficulty in this problem arises while prescribing the conformal class of the surface. This question comes naturally while trying to identify optimal conformal classes in connection with the so called Willmore conjecture. We will explain how this non local constraint is responsible for possible degeneracy of the a-priori elliptic problem to an hyperbolic one, the so called isothermic surfaces system. We will present some recent result about the geometric structure of the defect measure for sequences of isothermic surfaces.

Maxwell's equations and the Lorentz force

Laure Saint-Raymond
École Normale Supérieure, Paris, France

Using some refined compensated compactness arguments, we prove the stability of the Lorentz force when the electric and magnetic fields are governed by Maxwell's equations. As a consequence, we obtain some general existence result for MHD models. This is a joint work with Diogo Arsenio.

A priori estimates for 3D incompressible current-vortex sheets

Paolo Secchi
Università di Brescia, Italy

In this lecture we present a recent result about the free boundary problem for current-vortex sheets in ideal incompressible magneto-hydrody-namics. It is known that current-vortex sheets may be at most weakly (neutrally) stable due to the existence of surface waves solutions to the linearized equations. The existence of such waves may yield a loss of derivatives in the energy estimate of the solution with respect to the source terms.
However, under a suitable stability condition satisfied at each point of the initial discontinuity and a flatness condition on the initial front, we prove an a priori estimate in Sobolev spaces for smooth solutions with no loss of derivatives.
Our result gives some hope for proving the local existence of smooth current-vortex sheets without resorting to a Nash-Moser iteration. Such result would be a rigorous confirmation of the stabilizing effect of the magnetic field on Kelvin-Helmholtz instabilities, which is well known in astrophysics.
This is a joint work with Jean-François Coulombel, Alessandro Morando and Paola Trebeschi.

Conical waves in 3-D Riemann problems

Denis Serre
École Normale Supérieure, Lyon, France

The Riemann problem in d space dimensions leads naturally to study the interaction of d planar shock waves. Such an interaction is well understood in 2-D after the analysis by Courant and Friedrichs; it looks like a 1-D Riemann problem. Likewise, the case $d = 3$ looks like a 2-D Riemann problem, but with an initial data made of coherent waves. Using the example of irrotational gas dynamics, I'll emphasize the occurence of an elliptic zone, which corresponds to a conical wave. For a Chaplygin equation of state, I prove that the interaction problem admits a unique solution.

On the equations of nonlinear elasticity

Athanasios Tzavaras
University of Crete, Greece

The equations of elastodynamics are a paradigm of a system of conservation laws where the lack of uniform convexity of the stored energy function poses challenges in the mathematical theory. The existence of certain nonlinear transport equations for null-Lagrangeans reinforces the efficacy of the entropy as a stabilizing factor and recovers the strength associated wth uniformly convex entropies in hyperbolic systems. Elastodynamics with polyconvex stored energy can be embedded into a symmetric hyperbolic system and visualized as constrained evolution leading to a variational approximation scheme and an existence theory for measure valued solutions satisfying certain kinematic constraints in the weak sense. It provides a framework, in conjunction with the relative entropy method, to establish convergence of viscocity approximations or convergence of time-step approximants to smooth solutions of polyconvex elastodynamics. In addition, when a smooth solution is present it is unique within the class of measure valued solutions. The system of radial elastodynamics for isotropic elastic materials is an interesting case study. We present the format of the enlarged system in this special case with the objective of assigning a mechanical interpretation in the nonlinear transport constraints. It turns out that one can construct via variational approximation solutions that obeys the impenetrability of matter constraint.

Optimal control of discontinuous solutions of hyperbolic conservation laws: theory and numerical approximation

Stefan Ulbrich
Technische Universität Darmstadt, Germany

We consider optimal control problems governed by hyperbolic conservation laws. For the case of scalar convex conservation laws with source terms we present a sensitivity and adjoint calculus which implies the differentiability of a large class of objective functionals with respect to the control. Moreover, the gradient of the reduced objective funtional can conveniently be represented by an adjoint state that is the reversible solution of a transport equation with discontinuous coefficient. The approach has the potential to be extended to stable solutions of systems of conservation laws. Moreover, we analyse the convergence of discrete approximations of the considered optimal control problems. Building on the sensitivity and adjoint calculus we show that an appropriate discretization by finite difference schemes with sufficient numerical viscosity at shock discontinuities leads to convergent approximations of the linearized and the adjoint equation and thus to convergent gradient approximations of the cost functional. Hence, efficient optimization methods are applicable.

Time asymptotic stability of a viscous rarefaction wave

Shih-Hsien Yu
National University of Singapore

In this talk we review the pointwise approach to study the time asymptotic stability of a viscous rarefaction wave. This approach includes a primary approximation to the hyperbolic nonlinear wave, a construction of approximate Green's functions to a problem linearized around the approximate solution, and a separation procedure to extract the quadratic nonlinearity of Burgers' equation with a given inhomogeneous term.

INVITED TALKS

Recent developments in the theory of hyperbolic balance laws

Cleopatra Christoforou
University of Nicosia, Cyprus

In this talk, we will see some recent developments in the theory of hyperbolic balance laws related to the spreading of rarefaction waves and the convergence rate of the vanishing viscosity approximations. Further application of the theory of balance laws to differential geometry will be addressed.

An optimal harvesting problem with measure valued solutions

Giuseppe Coclite
Università di Bari, Italy

In this lecture we consider a model for the harvesting of marine resources, described by an elliptic equation. Since the cost functionals have sublinenar growth with respect to the pointwise intensity of fishing effort, optimal solutions are in general measure-valued. For the control problem, we prove the existence of optimal strategies. The results were obtained in collaboration with Professors Alberto Bressan and Wen Shen.

The Hamiltonian structure of the nonlinear Schrödinger equation and the asymptotic stability of its ground states

Scipio Cuccagna
Università di Trieste, Italy

We give an outline of the set up of the result on asymptotic stability of ground states of "generic" Nonlinear Schrödinger equations which has recently appeared in Comm.Math. Phys., 2011, Vol. 305, Number 2, Pages 279-331. Basically, the issue is that we have a dispersive system with both discrete and continuous components and we need to prove that the continuous ones dampen the discrete ones. This mechanism is often referred to as Fermi Golden Rule. It turns out that in the nonlinear setting what is crucial is the Hamiltonian structure of the NLS. In part it has been proved also for Dirac equations.

Analysis of defect measures and correctors for plasma oscillations

Donatella Donatelli
Università dell'Aquila, Italy

It is well known that a simplified model to describe the dynamics of a plasma is given by the the coupling of the compressible Navier Stokes equations with a Poisson equation , where in dimensionless units the coupling constant can be expressed in terms of a parameter λ which represents the scaled Debye length. The physical meaning of the Debye length λ is the screening distance or the distance over which the usual Coulomb field $1/r$ is killed off exponentially by the polarization of the plasma. In many cases the Debye length is very small compared to the macroscopic length $\lambda << 1$ and so it makes sense to consider the quasineutral limit $\lambda \to 0$ of the system. In this situation the particle density is constrained to be close to the background density (equal to one in our case) of the oppositely charged particle. The limit $\lambda \to 0$ is called the quasineutral limit since the charge density almost vanishes identically. The velocity of the fluid then evolves according to the incompressible Navier Stokes flow. This type of limit has been studied by many authors in the framework of smooth solution or well-prepared initial data. However there is no analysis for the quasineutral limit for the Navier Stokes Poisson system in the contest of weak solutions and in the framework of general ill prepared initial data. The common feature of this kind of limits in the ill prepared data framework is the high plasma oscillations, namely the presence of high frequency time oscillations of the acoustic waves. These fast waves complicate the mathematical analysis since they prevent the strong convergence of the gradient part of the velocity field. Our approach to avoid these difficulty is to estimate the behaviour of the acoustic waves by exploiting and combining together its dissipative and oscillating nature. Another issues which makes the limiting behaviour analysis very hard is the presence of very stiff terms due to the electric field E. By the a priori estimate we will only know L^2 estimates, which does not give enough information to pass into the limit in the quadratic terms appearing in our system. In order to recover the weak continuity of quadratic forms in L^2 we define the so called microlocal defect measure introduced by P. Gèrard and by L. Tartar, but in order to handle time oscillations we need to introduce correctors. They correspond to the physical phenomenon of the high frequency plasma oscillation. The correctors remain important as $\lambda \to 0$ and are not vanishing, in fact the effect of ill prepared initial data appears through the correctors and remains important for all times.

Asymptotic properties of linearized equations of low compressible fluid motion

Nikolay Gusev
Moscow Institute of Physics and Technology, Russia

The seminar will be devoted to linearized barotropic Navier-Stokes equations. An initial-boundary value problem for these equations will be considered. We will study existence and uniqueness of weak solutions to this problem and discuss the convergence of these solutions when compressibility tends to zero.

On representation of solution to scalar conservation law in several dimensions

Olga Rozanova
Moscow State University, Russia

We find an asymptotic representation of smooth solution to the Cauchy problem to scalar multidimensional conservation law. The representation is obtained by means of stochastic perturbation along characteristics. It helps, in particular, to study the process of singularities formation. Further, we introduce an associated system of conservation laws that can be interpreted as system of motion of continuum with some specific pressure term. This term arises only after the moment when he solution to the initial Cauchy problem looses smoothness. Before this moment the system coincides with the pressure free gas dynamics.
Moreover, we propose a certain iteration procedure that allows to find an approximation to the generalized solution to the Cauchy problem for arbitrary scalar conservation laws.

Zero-pressure gas dynamics as model of "dusty" media

Vladimir Shelkovich
St.Petersburg State University of Architecture
and Civil Engineering, Russia

We introduce integral identities to define δ-shock wave type solutions for the multidimensional zero-pressure gas dynamics

$$\begin{aligned} \rho_t + \nabla \cdot (\rho U) &= 0, \\ (\rho U)_t + \nabla \cdot (\rho U \otimes U) &= 0, \\ \Big(\frac{\rho |U|^2}{2} + H\Big)_t + \nabla \cdot \Big(\Big(\frac{\rho |U|^2}{2} + H\Big) U\Big) &= 0, \end{aligned}$$

where ρ is the density, $U \in \mathbb{R}^n$ is the velocity, H is the internal energy, $x \in \mathbb{R}^n$. To deal with such solutions, a specific analytical technique is developed. Using our definition, the corresponding Rankine-Hugoniot conditions are derived. In contrast to the case of shock waves, a δ-shock wave front *carries mass, momentum and energy.* We derive the balance laws describing mass, momentum, and energy transport from the area outside the δ-shock wave front onto this front. These processes are going on in such a way that the total mass, momentum, and energy are conserved and at the same time the mass and energy of the moving δ-shock wave front are increasing quantities. In addition, the total kinetic energy transforms into the total internal energy. The process of propagation of curvilinear δ-shock waves is also described. These results can be used in modeling of mediums which can be treated as a *pressureless continuum* (dusty gases, two-phase flows with solid particles or droplets, granular gases).

Stability estimates for the eigenvalues of the Dirichlet and Neumann Laplacian in rough domains

Laura V. Spinolo
University of Zurich, Switzerland
and CNR, Pavia, Italy

The talk will deal with quantitative estimates describing how the eigenvalues of a linear elliptic operator change when the domain is perturbed. I will be concerned with both cases of Dirichlet and Neumann boundary conditions and the estimates will hold for sets satisfying low regularity assumptions (namely, the so-called Reifenberg-flat domains). The talk will be based on a joint work with A. Lemenant and E. Milakis.