

Some Topics in Compressible Navier-Stokes Equations

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Abstract and Course Plan:

In this short course, we will concentrate on the well-posedness theory of multi-dimensional compressible Navier-Stokes system. We will begin by review some of the major progresses, and discuss some important open problems in this direction. In particular, we will describe the classical theory of global well-posedness of classical solutions which are suitably small perturbations of a uniform non-vacuum state, and the theory of global existence weak solutions of arbitrary amplitudes for isentropic compressible Navier-Stokes system. Then we will present in some details the local well-posedness theory of either strong or classical solutions to the 3-dimensional barotropic compressible Navier-Stokes system for general nonnegative density. Then some interesting blow-up phenomena will be presented in details, and we will then discuss also the recent studies of blow-up criterion of either strong or smooth solutions with arbitrary amplitudes. The role of the super-norm estimate on the density in the theory will be discussed. Then our focus will be on the global well-posedness of classical solutions to the 3-dimensional isentropic Navier-Stokes equations with small initial energy but possible large oscillations and vacuum. Though the analysis for such a theory is rather elementary, yet it generalizes the classical well-posedness of smooth non-vacuum solutions of Matsumura-Nishida to the important cases of large oscillations and vacuum, and yields the regularity and uniqueness of the weak solutions of P. L. Lions for small initial total energy. Details of the analysis will be presented.