Elastic deformations on the plane and approximations

(lecture IV)

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Plan of the course

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• Lecture I: *Mappings of finite distorsion and orientation-preserving homeomorphisms*.

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- Lecture IV: *The approximation result*.

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- Lecture V: *Bi-Lipschits extension Theorem (part 1)*.

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• Lecture V: *Bi-Lipschits extension Theorem (part 1)*.

• Lecture VI: *Bi-Lipschits extension Theorem (part 2)*.

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- v is CL^4 bi-Lipschitz for the (countably) piecewise affine approximation.
- v is $CL^{28/3}$ bi-Lipschitz for the smooth approximation.
- v is *finitely* piecewise affine whenever this makes sense.

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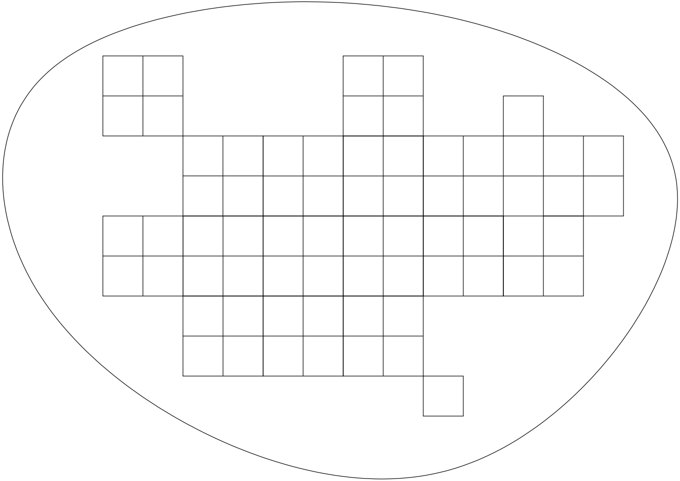

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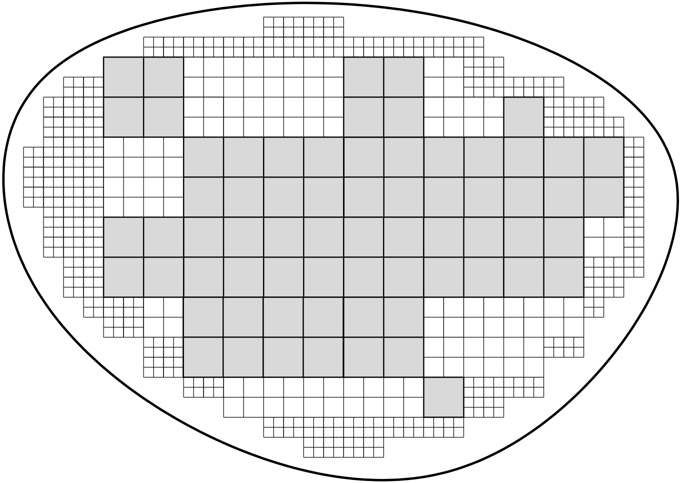

The proof of the result

- Definition of r-right polygon, r-piecewise affine function, tiling and r-interpolation.

- The L^∞ lemma.

- A large right polygon made by Lebesgue squares.

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- Good approximation of curves.
- Good approximation around the corners.

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