Elastic deformations on the plane and approximations

(lecture V–VI)

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Plan of the course

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- Lecture V: *Bi-Lipschits extension Theorem (part 1)*.

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• Lecture V: *Bi-Lipschits extension Theorem (part 1)*.

• Lecture VI: *Bi-Lipschits extension Theorem (part 2)*.

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Theorem (Daneri, P.): Let $u : \partial\mathcal{D} \to \mathbb{R}^2$ be piecewise affine and L bi-Lipschitz. Then there exists an extension of u which is CL^4 bi-Lipschitz.

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- If u is generic, then there is again a CL^4 bi-Lipschitz extension.

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Step I: Selecting the central ball.

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Step IV: Definition of the good paths.

Step V: Estimate on the length of the good paths.

The proof of the result (2/2)

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Step VI: Definition of the speed function.

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Step VII: The bi-Lipschitz extension on each primary sector.

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Step VII: The bi-Lipschitz extension on each primary sector.

Step VIII: The bi-Lipschitz extension in the internal polygon.

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Step VI: Definition of the speed function.

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Step VIII: The bi-Lipschitz extension in the internal polygon.

Step IX: The smooth extension.

The proof of the result (2/2)

Step VI: Definition of the speed function.

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Step VIII: The bi-Lipschitz extension in the internal polygon.

Step IX: The smooth extension.

Step X: The non piecewise affine case.

Thank you