FINITE ENERGY SOLUTIONS TO THE ISENTROPIC EULER EQUATIONS

Abstract. Considering the isentropic Euler equations of compressible fluid dynamics with geometric effects included, we establish the existence of entropy solutions for a large class of initial data. We cover fluid flows in a nozzle or in spherical symmetry when the origin $r = 0$ is included. These partial differential equations are hyperbolic, but fail to be strictly hyperbolic when the fluid mass density vanishes and vacuum is reached. Furthermore, when geometric effects are taken into account, the sup-norm of solutions can not be controlled since there exist no invariant regions. To overcome these difficulties and to establish an existence theory for solutions with arbitrarily large amplitude, we search for solutions with finite mass and total energy. Our strategy of proof takes advantage of the particular structure of the Euler equations, and leads to a versatile framework covering general compressible fluid problems. We establish first higher-integrability estimates for the mass density and the total energy. Next, we use arguments from the theory of compensated compactness and Young measures, extended here to sequences of solutions with finite mass and total energy. The third ingredient of the proof is a characterization of the unbounded support of entropy admissible Young measures. This requires the study of singular products involving measures and principal values.