Stability of finite difference schemes for hyperbolic initial boundary value problems

Jean-François Coulombel
CNRS & Université Lille 1, Laboratoire Paul Painlevé (CNRS UMR8524)
and Project Team SIMPAF of INRIA Lille Nord Europe
Cité scientifique, Bâtiment M2, 59655 VILLENEUVE D'ASCQ Cedex, France
Email: jfcoulom@math.univ-lille1.fr

This series of lectures will be devoted to the stability theory for fully discretized hyperbolic initial boundary value problems. Most of the results that we shall present deal with problems in one space dimension but we shall also indicate which parts of the theory easily extend to problems in several space dimensions. The major topic addressed in the lectures will be the characterization of stability for a numerical scheme by means of a suitable spectral condition that is analogous to the uniform Kreiss-Lopatinskii condition for hyperbolic partial differential equations.

The stability theory for fully discretized hyperbolic equations was initiated in the fundamental contribution by Gustafsson, Kreiss and Sundström (*Mathematics of Computation*, 1972). I shall first explain why, unfortunately, the theory developed by these authors can not cover all "reasonable" finite difference schemes, especially in view of a future extension to problems in several space dimensions. I shall then explain how this original theory can be generalized in order to cover a wider class of finite difference schemes. The analysis uses some ideas from the works by Métivier and Zumbrun (*Discrete and Continuous Dynamical Systems*, 2004, and *Journal of Differential Equations*, 2005). In the last part of the lectures, I shall review the derivation of semigroup estimates when one wishes to incorporate nontrivial initial data. The arguments there will not be restricted to the one-dimensional case.