Existence of algebraic vortex spirals and ill-posedness of inviscid flow

Volker Elling

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This short course will focus on spiral rollup of vortex sheets in incompressible flow. We will begin by studying the importance of this key problem for fluid mechanics as a whole, for the understanding of drag forces on solid objects immersed in fluid streams, but especially for non-uniqueness of the initial-value problem for inviscid flow models, as well as amplification of errors in viscous/kinetic perturbations and numerical approximation of such models. The relationship to the Kelvin-Helmholtz instability and other phenomena observed by Scheffer, Schnirelman, de Lellis/Székelyhidi, as well as Lopes/Lowengrub/Lopes/Zheng, is discussed as well.

After a brief survey of (few) prior analytical results on spiral rollup we will outline the overall approach for proving existence and asymptotic behaviour of self-similar vortex spirals, with scaling $x \sim t^\mu$. Two special cases, namely the limit of highly symmetric rollup and the limit of $\mu \downarrow \frac{1}{2}$ (corresponding to vorticity concentrating in the spiral center), are discussed in detail. In these two cases the spirals are close to conjectured asymptotic expansions of Kaden and Rott.

Finally, there will be a discussion of ongoing research on a general paradigm for constructing large-data cases, aiming in particular for an example of vortex sheet separation of Pullin (1989), but also the classical example of Kaden (1931).