On existence of global weak solutions for a class of quantum fluids systems

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During this talk we present some results obtained jointly with P. Marcati. We study existence of global weak solutions in the space of energy for a class of Quantum Hydrodynamics Systems. Here the main point is that we don't assume any further regularity and/or smallness assumptions (other than the energy to be finite) on the initial data. We achieve this goal by replacing the usual WKB formalism with a polar decomposition theory which is not limited by the presence of vacuum regions. In this way we set up a self consistent theory, based only on particle current densities, which does not need to define the velocity fields in the nodal regions. Furthermore we present some significant models to which these result apply.

Nonlinear wave equations with interior and boundary supercritical sources

Lorena Bociu
University of Nebraska

The talk will focus on the analysis and the influence of strong nonlinearities present in wave equations on bounded domains, and discuss the well-posedness of the system on the finite energy space. A distinct feature of the equation is the presence of the double interaction of supercritical sources and nonlinear damping terms, both in the interior of the domain and on the boundary. Moreover, the nonlinear boundary sources are driven by Neumann boundary conditions. Since Lopatinski condition fails to hold for dimension of the domain greater or equal than 2, the analysis of the nonlinearities supported on the boundary (within the framework of weak solutions) is a rather subtle issue and involves strong interaction between the sources and the damping terms. I will provide positive answers to the question of Hadamard well-posedness and moreover give complete and sharp description of parameters corresponding to global existence and blow-up of solutions in finite time.

On the Nonlinear Variational Wave Equation

Helge Holden
Norwegian University of Science and Technology

We prove existence of a global semigroup of conservative solutions of the nonlinear variational wave equation $u_{tt} - c(u)(c(u)u_x)_x = 0$. This equation shares many of the peculiarities of the HunterSaxton and the CamassaHolm equations. In particular, the equation possesses two distinct classes of solutions denoted conservative and dissipative. In order to solve the Cauchy problem it is necessary to augment the equation properly in order to obtain a unique solution. In this talk we describe how this is done for conservative solutions. The equation was derived by Saxton as a model for liquid crystals. The talk is based on joint work with X. Raynaud.

On stabilization and control for the critical Klein-Gordon equation on a 3-D compact manifold

Camille Laurent
Ecole Polytechnique, France

We study the internal stabilization and control of the critical nonlinear Klein-Gordon equation on 3-D compact manifolds. Under a geometric assumption slightly stronger than the classical geometric control condition, we prove exponential decay for some solutions bounded in the energy space but small in a lower norm. The proof combines profile decomposition and microlocal arguments. This profile decomposition, analogous to the one of Bahouri-Gérard on R3, is performed by taking care of possible geometric effects. It uses some results of S. Ibrahim on the behavior of concentrating waves on manifolds.

Controllability of a parabolic system with a diffusive interface

Matthieu Leautaud
Université Pierre et Marie Curie, France

We consider a linear parabolic transmission problem across an interface of codimension one in a bounded domain or on a Riemannian manifold, where the transmission conditions involve an additional parabolic operator on the interface. This system is an idealization of a three layer model, in which the central layer has a small thickness. We prove a Carleman estimate in the neighborhood of the interface for an associated elliptic operator by means of partial estimates in several microlocal regions. In turn, from the Carleman estimate, we obtain a spectral inequality that yields the null-controllability of the parabolic system. These results are uniform with respect to the small thickness parameter. This is a joint work with Jrme Le Rousseau and Luc Robbiano.

Existence of strong solutions to fluid-structure systems

Julien Lequeurre
Université Paul Sabatier, France

We study coupled fluid-structure systems in two and three dimensions. The structure corresponds to a part of the boundary of a domain containing an incompressible viscous fluid. The structure displacement is modeled either by a damped beam equation or a strongly damped wave equation. We prove for each system the existence of strong solutions for small data and the existence of local strong solutions for any initial data.

Lower compactness estimates for scalar balance laws

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We study the compactness in L^1_{loc} of the semigroup $(S_t)_{t\geq 0}$ of entropy weak solutions to strictly convex scalar conservation laws in one space dimension. The compactness of S_t for each $t > 0$ was established by P. D. Lax [1]. Upper estimates for the Kolmogorov's ε-entropy of the image through S_t of bounded sets C in $L^1 \cap L^\infty$ which is denoted by

$$H_\varepsilon(S_t(C) \mid L^1(\mathbb{R})) := \log_2 N_\varepsilon(S_t(C)).$$

where $N_\varepsilon(S_t(C))$ is the minimal number of sets in a cover of $S_t(C)$ by subsets of $L^1(\mathbb{R})$ having diameter no larger than 2ε, were given by C. De Lellis and F. Golse [3]. Here, we provide lower estimates on this ε-entropy of the same order as the one established in [3], thus showing that such an ε-entropy is of size $\approx (1/\varepsilon)$. Moreover, we extend these estimates of compactness to the case of convex balance laws.
This is a joint paper with F. Ancona and O. Glass.

References

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3. De Lellis C., Golse F., *A Quantitative Compactness Estimate for Scalar Conservation Laws*, Comm. Pure Appl. Math. 58 (2005), no. 7, 989–998.

Asymptotic stabilization of entropy solutions to scalar conservation laws through a closed feedback loop

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We consider the initial boundary value problem for a scalar conservation law with a $\mathcal{C}^2$ flux f strictly convex:

$$\begin{gathered} u_t + (f(u))_x = g(t) \quad \text{on} \quad \mathbb{R}^+ \times (0,1), \\ \text{"}u(t,0) = u_l(t)\text{"}, \quad \text{"}u(t,1) = u_r(t)\text{"} \quad \text{for} \quad t \in \mathbb{R}^+, \\ u(0,.) = u_0 \quad \text{on} \quad (0,1). \end{gathered} \tag{1}$$

It was shown in [1] and [2] that we should not request the equality everywhere on the boundary hence the quotation marks. Now for any constant $\bar{u}$, if we consider the closed loop law:

$$\begin{aligned} g = \alpha ||u(t,.) - \bar{u}||_{L^1(0,1)} \\ u_l = \bar{u}, \\ u_r = \bar{u}, \end{aligned} \tag{2}$$

then for α sufficiently small, we show that for any u_0 in $\mathrm{BV}(0,1)$ there exists a unique entropy solution u of (1) and (2) in the sense of [1]. Furthermore we have the following asymptotic stabilization property:

$$||u(t,.) - \bar{u}||_{L^1(0,1)} \to 0 \quad \text{when} \quad t \to +\infty. \tag{3}$$

References

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2. Leroux A. Y., *Etude du probleme mixte pour une equation quasi-lineaire du premier ordre*, C. R. Acad. Sci. Paris, serie A, T. 285 (1977), pp. 351-354.