Long time behavior and its control in nonlinear hyperbolic-like evolutions.

I. Lasiecka
University of Virginia
Department of Mathematics
Charlottesville, VA

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These lectures are devoted to the analysis of stability and control of long time behavior of PDE models described by nonlinear evolutions pf hyperbolic type. Specific examples of the models under consideration include: (i) nonlinear systems of dynamic elasticity: von Karman systems, Berger's equations, Kirchhoff - Boussinesq equations, nonlinear waves (ii) nonlinear structure-flow interactions, (iii) and nonlinear thermo-elasticity. A goal to accomplish is to reduce the asymptotic bahavior of the dynamics to a tractable finite dimensional and possibly smooth sets. Then the methods of finite dimensional control theory can be used in order to forge a desired outcome for the system.

A characteristic feature of the models under consideration is criticality or super-criticality of sources (with respect to Sobolev's embeddings) along with supecriticality of damping mechanisms which may be also geometrically constrained. This means the actuation takes place on a "small" region only. Supercriticality of the damping is often a consequence of the "rough" behavior of nonlinear sources in the equation. Controlling supercritical potential energy may require a calibrated nonlinear damping that is also supercritical. On the other hand super-linearity of the potential energy provides beneficial effect on the long time boundedness of semigroups. From this point of view, the nonlinearity does help controlling the system but, at the same time, it also does raise a long list of mathematical issues starting with a fundamental question of uniqueness and continuous dependence of solutions withe respect to the given (finite energy) data. It is known that solutions to these issues can not be handled by standard nonlinear analysis-PDE techniques. The aim of these lectures is to present several methods of nonlinear PDE which include cancellations, harmonic analysis and geometric methods which enable to handle super-criticality in both sources and the damping. It turns out that if carefully analyzed the nonlinearity can be taken "advantage of" in order to produce implementable control algorithms.

Another aspects that will be considered is the understanding of control mechanisms which are geometrically constrained. Here one would like to use minimal sensing and minimal actuating (geometrically) in order to achieve the prescribed goal. This is indeed possible, however analytical methods used are more subtle. The final task boils down to showing that passively controlled system is "quasi-stable" i.e attracted at the uniform rate to a compact set. Showing this property- formulated as quasi-stability estimate -is the key and technically demanding issue. Much of the lecture time will be devoted to development of suitable tools for proving quasi-stability. These include tools such as weighted energy inequalities, compensated compactness, Carleman's estimates and microlocal analysis.

1. Presentation of several PDE control models and a general discussion of the role of the source and the damping and their interaction as a mechanisms of control.

2. Wellposedness of control systems including control-theoretic properties of the control-observation maps. Controllability, observability, stabilization with full and partial observations.

   - Cancellations-harmonic analysis and microlocal analysis methods
   - Sharp control of Sobolev's embeddings and duality scaling.
   - Role of superllinear potential energy and superlinear damping

3. General tools for studying attractors

   - Absorbing balls and attractors by weighted energy methods.
   - Quasi-stability Inequality and its consequences. Finite dimensionality of attractors, smoothness of attractors.
   - Exponential attarctors and controlled decay rates to the equilibria.

4. How to prove quasi-stability?

   - Gradient systems and non-gradient systems.
   - Interior nonlinear damping.
   - Geometrically constrained damping. Carleman estimates, topological methods, backward uniqueness, backward regularity.

5. PDE illustrations.

   - Nonlinear waves and plates with nonlinear damping and supercritical sources.
   - Nonlinear waves and plates with geometrically constrained damping.
   - Thermal-structure interactions.
   - Flow-structure interactions.
   - Fluid-structure interactions

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