On the definition of Sobolev spaces in metric measure spaces

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I will discuss the definition of the Sobolev space $W^{1,2}$ on a general metric measure space, which amounts in finding an appropriate notion of 'modulus of weak gradient'. There are two possible approaches:

- relaxing the local Lipschitz constant;
- looking for a minimal function which is an upper gradient along "almost any curve".

I will show that these two approaches coincide, regardless of any assumption on the metric measure space (beside some exponential control on the volume of concentric balls). Previous results on the subject required the measure to be doubling and the validity of a local Poincaré inequality.
From a collaboration with L.Ambrosio and G.Savaré.