Controllability of some coupled non linear parabolic equations

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Université de Provence, France

This talk concerns the question of controllability of n coupled non linear parabolic equations by $m < n$ distributed controls. The main difficulty is to deal with less control than the number of equations. We will focus on the case where $n = 2$ and $m = 1$ and consider a general reaction-diffusion system which arises in mathematical biology :

$$\begin{cases} \psi_t = \Delta\psi + f_1(\psi, w) \ \text{ in } Q_T = \Omega \times (0,T) \\ w_t = \Delta w \ + f_2(\psi, w) \ + \chi_\omega \, g \text{ in } Q_T \\ \psi = w = 0 \ \text{ on } \Sigma_T = \partial\Omega \times (0,T) \\ \psi(x,0) = \psi_0, \, w(x,0) = w_0, \ x \in \Omega \end{cases} \tag{1}$$

where Ω is a bounded domain of $\mathbb{R}^n$with smooth boundary $\partial\Omega$, f_i $(i = 1, 2)$ are smooth real functions (let us say C^2 functions) and g is a control in $L^2(Q_T)$. Let g^* in $L^2(Q_T)$ (with $Q_T = \Omega\times]0,T[$), and $(\psi_0^*, w_0^*) \in L^2(\Omega)^2$. Suppose that there exists a (ψ^*, w^*) satisfying (1) in $C\left([0,T] \times L^2(\Omega)\right)^2$with $(\psi(0), w(0)) = (\psi_0^*, w_0^*)$. Therefore, by setting

$$\begin{aligned} \psi &= \overline{\psi} - \psi^* \\ w &= \overline{w} - w^* \end{aligned}$$

where $(\overline{\psi}, \overline{w}, \overline{g})$ satisfies (1), one gets:

$$\begin{cases} \psi_t = \Delta\psi + f_1(\overline{\psi}, \overline{w}) - f_1(\psi^*, w^*) \ \text{ in } Q_T \\ w_t = \Delta w \ + f_2(\overline{\psi}, \overline{w}) - f_2(\psi^*, w^*) + \chi_\omega g \ \text{ in } Q_T \\ \psi = w = 0 \ \text{ on } \Sigma_T \\ \psi(x,0) = \psi_0, \, w(x,0) = w_0, \ x \in \Omega \end{cases} \tag{2}$$

where $g = \overline{g} - g^*$. Our aim is, for any (ψ_0, w_0) belonging to a suitable space, to find a control $g \in L^2(Q_T)$ such that the associated solution of (2) satisfies

$$\psi(T) = w(T) = 0 \text{ on } \Omega \tag{3}$$

This is the null-controllabillity property.

Theorem 1 *Assume that $f_i \in C^1(\mathbb{R}^2, \mathbb{R})$ for $i = 1, 2$, and let $T > 0$.*

(i) Global controllability to the trajectories: *If for $i = 1, 2$, f_i satisfies:*

$$\lim_{(|s|+|r|)\to+\infty} \frac{f_i(r,s)}{\ln^{3/2}(1 + |s| + |r|)} = 0 \ . \tag{4}$$

and there exists $\mu > 0$ such that:

$$\frac{\partial f_1}{\partial s}(r,s) \geq \mu, \, \forall (r,s) \in \mathbb{R}^2. \tag{5}$$

Then for all $\psi_0, w_0 \in H_0^1(\Omega) \cap W^{2(1-\frac{1}{q_N}),q_N}(\Omega)$*, one can find* $g \in L^{q_N}(Q_T)$ *such that there exists* (ψ_g, w_g) *solution of (1) with* $\psi_g, w_g \in W^{2,1}_{q_N}(Q_T)$ *and satisfying:*

$$\psi_g\ (T) = 0,\ \ w_g\ (T) = 0.$$

(ii) Local controllability to the trajectories: *If* f_1 *satisfies (5), there is* $\rho > 0$ *such that if* $\psi_0\ , w_0\ \in H_0^1(\Omega) \cap W^{2(1-\frac{1}{q_N}),q_N}(\Omega)$ *with*

$$\|(\psi_0\ , w_0\ )\|_{L^\infty(\Omega)} \le \rho\,,$$

one can find $g\ \in L^{q_N}(Q_T)$ *such that there exists* (ψ_g, w_g) *solution of (1) with* $\psi_g\ , w_g\ \in W^{2,1}_{q_N}(Q_T)$ *and satisfying:*

$$\psi_g\ (T) = 0,\ \ w_g\ (T) = 0.$$

We will also give some new generalizations of this kind of results for more general parabolic systems.

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Control of coupled systems of PDE's by a reduced number of controls or feedbacks

Fatiha Alabau-Boussouira
Université Paul Verlaine-Metz

The purpose of this talk is to present several aspects and results on the stabilization, exact controllability and observability of coupled systems of PDE's.
When dealing with systems involving quantities described by several components, it might be impossible to control or observe all the state variables. Such limitations appear in applications to mathematical models for the vibrations of exible structures, electromagnetism or fuid control. Therefore it becomes more and more important to control coupled systems by a reduced number of controls. The purpose of this talk is to present several results on these questions.

The LQ-problem for evolutionary PDE with boundary control: classical problem, renewed challenges

Francesca Bucci
Università di Firenze

In this talk we shall discuss the infinite time horizon Linear-Quadratic problem for a class of abstract systems – introduced in 2005 by Acquistapace et al., initially inspired by thermoelastic plate equations with thermal control on the boundary – that encompasses significant Partial Differential Equations systems comprising both hyperbolic and parabolic components. In spite of being a classical problem, the LQ-problem continues to bring about novel technical challenges (when it pertains to composite systems of PDE), which arise on a theoretical level in the study of well-posedness of the corresponding Riccati Equations, and then in the analysis to be carried out on specific PDE models, in order to establish the sought regularity properties of boundary traces. (Most results presented in the talk are obtained jointly with Paolo Acquistapace (Pisa) and Irena Lasiecka (Virginia).)

Cardiovascular stents as nets/networks of hyperbolic balance laws

Suncica Canic
University of Houston

This talk will present an introduction to a new framework for modeling the metallic mesh-like structures such as cardiovascular stents, which can be described by a net/network of hyperbolic balance laws. By using dimension reduction the slender stent struts that make up the stent mesh,

are approximated by a family of 1D curved rods modeled by the Antman-Cosserat model that is in the form of a system of hyperbolic balance laws. Using multi-scale approaches, the slender curved rods are combined into a 3D hyperbolic stent net via certain geometric conditions, and via the contact conditions describing the physics of the interaction between the rods.
This approach is in contrast with the classical approaches in which a stent is modeled as a single, 3D elastic body, using commercial software packages based on 3D Finite Element Method structure approximations. Simulating slender stent struts using 3D approaches is computationally very expensive, typically producing simulation results with poor accuracy due to the insufficient mesh refinement imposed by the large memory requirements associated with the use of 3D meshes to approximate the slender stent struts.
Numerical simulation of the novel hyperbolic net problem that will be discussed in this talk is simple, and substantially cheaper than the 3D approaches, providing a manageable and simple framework for the studies of stents mechanical properties and for the studies of control and optimal stent design.
A comparison between the numerical solutions of the full 3D model and the solution of the novel 1D model will be presented, showing that the 3D solutions approach, with 3D mesh refinement, the solution of the novel 1D model. Application to the study of the mechanical properties of 4 currently available coronary stents on the US market, will be shown.
Due to the fact that the hyperbolic theory of network problems for systems of balance law is under-developed, understanding the behavior of solutions to the class of mathematical problems discussed in this talk will shed light not only on the mechanical properties of stents and the related structures, but also on the structure of solutions to hyperbolic nets and networks in general.
Collaborators include: Prof. Josip Tambaca (Mathematics, University of Zagreb, Croatia), Dr. David Paniagua (Cardiovascular Catheterization Laboratory, Michael E. DeBakey Veterans Medical Center, Baylor College of Medicine, and Texas Heart Institute, Houston), and student Mate Kosor (UH and University of Zagreb).

Null controllability for some classes of degenerate parabolic operators

Piermarco Cannarsa
Università di Roma "Tor Vergata"

Degenerate parabolic operators play an important role in both theoretical and applied domains. From the point of view of control theory, they present analogies as well as striking differences with respect to uniformly parabolic equations. The talk will discuss such phenomena for operators in two space dimension that degenerate either at the boundary of the space domain, or on ‘small’ subsets of the interior.

Conservation laws: modeling, analysis and control

Rinaldo M. Colombo
Università di Brescia, Italy

This presentation overviews models based on conservation laws that were recently introduced. First, the main modeling assumptions are justified, then the key analytical properties are presented while the qualitative behaviors of solutions are described through numerical integrations. Each of these models naturally leads to various optimal control problems. In this framework, both existence results and open problems are presented.

A patchy dynamic programming method for the numerical solution of Hamilton-Jacobi equations

Maurizio Falcone
Università di Roma “La Sapienza”

We present an approximation method for the solution of Hamilton-Jacobi equations which combines a patchy decomposition of the domain and a dynamic programming scheme. The patchy vector fields leading to the decomposition are inspired by a class of piecewise smooth vector fields introduced by Ancona and Bressan to study feedback stabilization problems. Since the subdomains are invariant with respect to the patchy vector fields, we can split the computation of the solution in each sub-domain and use a parallel algorithm to compute the value function in the whole domain. The patchy method has shown to be more efficient with respect to standard domain decomposition techniques as we will illustrate by some numerical tests.
Work in collaboration with: S. Cacace, E. Cristiani, A. Picarelli (Università di Roma “La Sapienza”)

Domain sensitivity in singular limits of compressible viscous fluids

Eduard Feireisl
Institute of Mathematics, Praha

We study the incompressible limit of the Navier-Stokes system on condition that the shape of the domain may vary with the Mach number. The limit system as well as the limit boundary conditions are identified and certain robustness of the method shown on unbounded domains with slip boundary conditions.

On the movement of a rigid body
in a perfect incompressible fluid

Olivier Glass
Université Paris-Dauphine

Let us consider the movement of a rigid body in a perfect incompressible fluid modeled by the Euler equations, where the body is driven by the pressure forces on its boundary. I will discuss several aspects of this system: representation as a geodesic flow, regularity

On the exact controllability to trajectories of the
nonlinear heat equation in unbounded domains

Manuel Gonzalez-Burgos
Universidad de Sevilla, Spain

In this talk we will consider the nonlinear heat equation

$$\begin{cases} \partial_t y - \Delta y + f(y) = v 1_\omega \text{ in } Q = \Omega \times (0,T), \\ y = 0 \text{ on } \Sigma = \partial\Omega \times (0,T), \quad y(\cdot,0) = y_0 \text{ in } \Omega, \end{cases} \tag{1}$$

where $\Omega \subseteq \mathbb{R}^N$ is a domain (bounded or not) with a regular boundary $\partial\Omega$, $\omega \subset \Omega$ is an open subset and $T > 0$. We will assume that $f : \mathbb{R} \to \mathbb{R}$ is a C^1 function and y_0 is given in an appropriate space.
It is well known that for obtaining the exact controllability to trajectories for the previous system, it is crucial the construction of a control v in $L^\infty(Q)$ (with estimates of the norm with respect to the data) which provides the null controllability result for the linear problem

$$\begin{cases} \partial_t y - \Delta y + ay = v 1_\omega \text{ in } Q, \\ y = 0 \text{ on } \Sigma, \quad y(\cdot,0) = y_0 \text{ in } \Omega, \end{cases} \tag{2}$$

($a \in L^\infty(Q)$ is given).
In this talk we will obtain the exact controllability to trajectories of system (1) in the case in which Ω is an unbounded open set, ω satisfies some geometrical conditions and the nonlinearity f has a slight superlinear growth at infinity.

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Control of Transient Gas Networks

Martin Gugat
Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany

Consider a gas transmission system with a network of gas pipelines. The flow through each pipe is governed by a hyperbolic system of partial differential equations where the source term that models the friction at the boundary is essential. At each vertex of the network graph the flows are coupled by algebraic node conditions. The flow is controlled by compressor stations.
We present results on stabilization by boundary feedback and on exact boundary controllability for the flow through such a gas pipeline network.

On the uniform L^1-stability of the Boltzmann and Vlasov-Poisson equations

Seung-Yeal Ha
Seoul National University

In this talk, I will present a recent progress on the uniform L^1-stability and instability of the two prototype kinetic equations (the Boltzmann and Vlasov-Poisson(V-P) system). For the V-P system in high dimensions ($d \geq 4$), the small amplitude decaying solutions are uniformly L^1-stable due to the strong dispersion property of the corresponding linearized equation, while for three dimensions, such an uniform L^1-stability estimate for the V-P system is still not known even for small solutions. I will report two negative results on the non-existence of L^1-asymptotic completeness and instability of compacton solution for the V-P system in three dimensions, which might suggest the possible scenario for the L^1-instability of the Vlasov-Poisson system in three dimensions. This is a joint work with Sun-Ho Choi (SNU).

Numerical optimization with hyperbolic conservation laws

Michael Herty
RWTH Aachen University

We present some analysis and numerical results for numerical methods for solving optimization problems with constraints given by hyperbolic differential equations. We focus on the numerical treatment using relaxation schemes. The later are well-known and easy to implement discretization schemes for systems of conservation or balance laws. Hereby, the original (nonlinear) system of balance laws is replaced by a linear system of double size, called the relaxation system. Using asymptotic analysis it can be shown that the relaxation system is well-posed if the new system matrix satisfies the so-called subcharacteristic condition. Under this assumption a solution to the relaxation system is known to converge to a solution of the original system. Furthermore, using IMEX-schemes for the time integration of the numerical scheme, it can be shown that the discretized relaxation system converges to a discretization of the limit equations. To provide consistent schemes for optimal control, we derive conditions such that the discrete adjoint of the relaxation system is a valid discretization of the continuous adjoint relaxation system in the context of smooth solutions. Furthermore, we prove that the discretization of the adjoint relaxation system converges to a discretization of the adjoint limit equation. This discretization then turns out to be the adjoint of the discretized limit equation.

Lagrangian controllability of the Euler equations

Thierry Horsin
Université de Versailles

I will describe common works with O. Glass and works in progress with O. Glass, G. Legendre and O. Kavian on the controllability of the motion of a set of fluid particles, describing the theoretical results and the possible use of them in order to perform computations.

Carleman estimates and inverse problems for some dynamical system from anisotropic elasticity theory

Victor Isakov
Wichita State University, USA

We derive Carleman type estimates for the system of elasticity with residual stress and apply them to obtain uniqueness and stability of the continuation results and to identification of the elastic parameters. We obtain Hölder and Lipschitz type stability estimates for various boundary data. A technical tool is Carleman estimates with two large parameters for general partial differential operators of second order.

Chemotaxis - from kinetic equations to aggregate dynamics

François James
Université d'Orlèans, France

We start from a kinetic model for chemotaxis, in which appears a characteristic time tending to zero. Formally it is quite easy to write the corresponding hydrodynamic limit as a nonlinear conservation law coupled with an elliptic equation. Concentration occurs (Dirac masses), which is the manifestation of the aggregation of bacteria. Several mathematical problems arise then, around the proper definition of the flux in the conservation law. An adaptation of the notion of duality solutions eventually allows to obtain the dynamics of Dirac masses–aggregates.

Geometric aspects of controllability for a 3-D swimmer's model

Alexander Khapalov
Washington State University

We discuss a 3-D coupled nonlinear pde model of a flexible swimmer consisting of a sequence of connected parallelepipeds (thin plates), which is placed in a nonstationary fluid. It is derived from the "immerse boundary" modeling approach due to C.S. Peskin. The resulting infinite dimensional evolution system is governed by internal swimmer's forces regarded as multiplicative controls. We investigate how the geometric shape of the parts of swimmer's body affects the forces acting upon it in a fluid.
This work is a step in an extension of our 2-D theory for the swimming phenomenon (modeling, wellposedness and controllability), presented in [Khapalov, *Lectures Notes in Mathematics*, Vol. 1995, Springer, 2010] and the references therein, to the 3-D case.
These new results are a collaborative effort with Giang-Bang Trihn.

Controlling short and long time behavior in Kirchhoff-Boussinesq equations.

Irena Lasiecka
University of Virginia

We shall consider evolutionary systems associated with nonlinear Kirchhoff-Boussinesq models. These nonlinear plate models are characterized by the presence of a nonlinear source that alone leads to finite time blow up of solutions. In order to counteract, control restorative forces are introduced. These internal forces, by necessity , are of supercritical nature. This raises natural questions such as : (1) uniqueness of weak solutions, (2) their regularity and (3) long time behavior. By using recently developed methods based on topological rescaling, harmonic analysis and sharp control of Sobolev's embeddings full Hadamard well-posedness for weak -energy based-solutions will be shown.
Long time behavior and existence of global attractors depend on the presence of the frictional damping in the model. It will be shown that for sufficiently large values of frictional damping parameter solutions not only exist globally but they are also attracted by a finite dimensional and smooth attractor. This result is obtained by applying the theory of quasi-stable dynamical systems. Several related questions such as whether long time behavior can

be controlled by geometrically constrained or nonlinear dissipation will also be discussed.

Functional inequalities and Hamilton-Jacobi equations

Paola Loreti
Università di Roma "La Sapienza"

Here we discuss some recent results on some functional inequalities related to a class of Hamilton-Jacobi equations. The talk is based on a joint work with A. Avantaggiati and C. Pocci.

Vortex layers of small thickness

Marco Sammartino
Università di Palermo, Italy

In this talk we shall treat the case of a planar inviscid flow with initial datum of vortex layer-type: vorticity of $\mathcal{O}(\varepsilon^{-1})$ is concentrated on a layer of thickness ε in such a way that, in the limit $\varepsilon \to 0$, the vorticity distribution converges to a δ-function concentrated on a curve.
In an analytic functional setting we shall prove that the Euler equation are well-posed for a time that does not depend on the thickness of the layer. This is a generalization of the result of D.Benedetto and M.Pulvirenti, SIAM J. Appl. Math. 52 (1992), where the authors considered the case of vortex layers of uniform vorticity.
We shall also discuss the possibility of using our result as a step toward the justification of the Birkhoff-Rott equation starting from the Navier-Stokes equations.
This is a joint work with M.C. Lombardo (Università di Palermo).

Density constraints in crowd evolutions

Filippo Santambrogio
Université Paris-Sud

Besides microscopic models describing crowds as a collection of small disks subject to non-overlapping constraints, continuous models may be considered, using densities and transforming the constraints into $\rho \leq 1$. In some papers with Maury and Roudneff-Chupin we described the evolution, mainly in a gradient flow setting, under the key idea that "the velocity fields that actually advects the motion is the projection of the desired one on the set of admissible fields", where admissible fields are those that have negative divergence on the set where $\rho = 1$. The same idea may be adapted to other settings and, after presenting the existence result for the "spontaneous" evolution, I will also present how to insert controls or strategic issues (like in mean-field games).

Controllability of cubic Schroedinger equation via low-dimensional source term

Andrey Sarychev
Università di Firenze

In the talk we present results on controllability of 2D defocusing cubic Schroedinger equation under periodic boundary conditions with control applied via source term. The source term is linear combination of few complex exponentials (modes) with time-variant coefficients - controls. We manage to prove that controlling just 4 modes one can achieve controllability of cubic Schroedinger equation in any finite-dimensional projection of its evolution space $H^2(\mathbb{T}^2)$, as well as approximate controllability in $H^2(\mathbb{T}^2)$. We also present a negative result regarding exact controllability of Schroedinger equation via finite-dimensional source term.

Two phase flow in porous media: undercompressive shocks and fingering instability

Michael Shearer
North Carolina State University

The one-dimensional Buckley-Leverett equation for immiscible two phase flow in a porous medium is a scalar conservation law with dissipation due to equilibriated capillary pressure. Consequently, traveling waves approximate shocks satisfying the classical Lax entropy condition. When the capillary pressure is modified to allow rate dependence, undercompressive shocks appear, and Riemann problems have nonclassical solutions that are visualized in numerical simulations.
However, all these shocks are typically unstable due to a subtle two-dimensional effect first analyzed for an idealized hyperbolic-elliptic system by Saffman and Taylor. We give a more comprehensive analysis of linearized stability along the lines of the long-wave expansion of Yortsis and Hickernell, identifying sufficient conditions for the Saffman-Taylor fingering instability. This is a joint work with Kimberly Spayd (North Carolina State University).

An abstract semi-group approach to the third-order M-G-T PDE-equation of high-intensity ultrasound

Roberto Triggiani
University of Virginia

The Moore-Gibson-Thompson equation arising in high intensity ultrasound is a third order equation (in time): it consists of a linear and a non-linear part. An abstract semi-group approach will be given to analyze the linear part. It provides: generation of a sc group; an interesting structural decomposition; spectral analysis; and exponential stability. The results are explicit

and precise (and computable) in terms of the data of the problem. They are next illustrated and confirmed by a computer-based analysis. In particular, the generator does not have compact resolvent. Finally, in the case one critical coefficient is space dependent, we shall solve the corresponding inverse problem (both uniqueness and Lipschitz-stability) of recovering such coefficients by using one additional measurement on a suitably identified, explicit portion of the boundary.
This a joint work with R.Marchand, T. McDevitt.