IMSE 2016

The 14th International Conference on

Integral Methods in Science and Engineering

Department of Mathematics,
University of Padova, 25 - 29 July 2016

Book of Abstracts

Contents

The International Conference on Integral Methods in Science and Engineering

The series of conferences on Integral Methods in Science and Engineering is an established forum where scientists from all over the world discuss their latest developments in original and effective methodologies relevant for solutions of mathematical models that describe physical phenomena, processes and related subjects.

Participation is open to all scientists and engineers whose work makes use of analytic and numerical methods, integral equations, ordinary and partial differential equations, asymptotic and perturbation methods, boundary integral techniques, stochastic methods, symmetries and conservation laws, hybrid approaches, vortex methods, signal processing and image analysis, among others.

One of the objectives is to promote new research tools, methods and procedures beyond the specific realms of Mathematics, the individual Exact or Technological Sciences besides their technological applications. Thus, the scope of the conference addresses academical and industrial interests.

The conference will be organized by the University of Padova in association with the University of Tulsa, Oklahoma, USA.

The official language of the conference is English.

Conference policies: the IMSE conference will be conducted in accordance with the International Union of Pure and Applied Physics (IUPAP) principles as stated in the IUPAP resolution passed by the General Assembly in 2008. In particular, no bona fide scientist will be excluded from participation on the grounds of national origin, nationality, or political considerations unrelated to science.

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SECTION 1

Plenary Talks

Resonances for plasmonic nanoparticles

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Localized surface plasmons are charge density oscillations confined to metallic nanoparticles. Excitation of localized surface plasmons by an electromagnetic field at an incident wavelength where resonance occurs results in a strong light scattering and an enhancement of the local electromagnetic fields. This talk is devoted to the mathematical modeling of plasmonic nanoparticles. Its aim is fourfold: (i) to mathematically define the notion of plasmonic resonance and to analyze the shift and broadening of the plasmon resonance with changes in size and shape of the nanoparticles; (ii) to study the scattering and absorption enhancements by plasmon resonant nanoparticles and express them in terms of the polarization tensor of the nanoparticle. Optimal bounds on the enhancement factors are also derived; (iii) to show, by analyzing the imaginary part of the Green function, that one can achieve super-resolution and super-focusing using plasmonic nanoparticles, and (iv) to establish an effective medium theory for resonant plasmonic systems.

Based on joint works with Pierre Millien (EPFL, Switzerland), Matias Ruiz (ENS Paris, France), Hai Zhang (UST, Hong Kong), and Sanghyeon Yu (ETHZ, Switzerland).

Keywords: super-resolution, plasmonic resonances, effective medium theory, Neumann - Poincaré operator, layer potential techniques.

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The Simple Layer Potential Approach to the Dirichlet Problem: an Extension to Higher Dimensions of the Muskhelishvili Method and Applications

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If we try to solve the Dirichlet problem for Laplace equation in a bounded domain Ω by means of a simple layer potential, we obtain an integral equation of the first kind on $\partial\Omega$. Muskhelishvili, in his famous book on singular integral equations [1], considering the bidimensional case, gave a method for solving this equation. His method, hinging on the theory of holomorphic functions of one complex variable, can not be easily extended to higher dimensions.

Some time ago an extension to multidimensional problems of the Muskhelishvili method was proposed in [2]. The aim of this talk is to present this method and to show some more recent applications to other problems (see [3] and the reference therein).

Based on joint work with V. Leonessa, A. Malaspina.

Keywords: potential theory, indirect methods, integral representations.

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A Qualitative Approach to Inverse Scattering Theory for Inhomogeneous Media: The Transmission Eigenvalue Problem

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The field of inverse scattering theory has been a particularly active field in applied mathematics for the past thirty years. The aim of research in this field has been to not only detect but also to identify unknown objects through the use of acoustic, electromagnetic or elastic waves. Until a few years ago, essentially all existing algorithms for target identification were based on either a weak scattering approximation or on the use of nonlinear optimization techniques. A survey of the state of the art for acoustic and electromagnetic waves can be found in [1]. However, as the demands of imaging increased, it became clear that incorrect model assumptions inherent in weak scattering approximations imposed severe limitations on when reliable reconstructions were possible. On the other hand, it was also realized that for many practical applications nonlinear optimization techniques require a priori information that is in general not available. Hence, in recent years, alternative methods for imaging have been developed that avoid incorrect model assumptions but, as opposed to nonlinear optimization techniques, only seek limited information about the scattering object. Such methods come under the general title of qualitative methods in inverse scattering theory [2] and are based on the analysis of a linear integral equation of the first kind with smooth kernel called the far field equation. In general a solution to the far field equation does not exist, although uniqueness holds provided the wave number is not a transmission eigenvalue. In this talk we will give a survey of this rapidly growing area of inverse scattering theory with particular attention on giving meaning to a regularized solution of the far field equation as well as the role that transmission eigenvalues play in determining the material properties of the scattering object. We will focus our attention on the scattering of acoustic waves by an inhomogeneous medium.

Keywords: inverse scattering, linear sampling method, far field operator, transmission eigenvalues.

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CONVERGING EXPANSIONS FOR SELF-SIMILAR PERFORATIONS OF A PLANE SECTOR

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The perforation pattern Q is the union of a finite number of Lipschitz connected domains. For $\varepsilon > 0$ small enough, the perforated sector A_ε is a bounded sector A_0 with vertex $\{0\}$, from which is removed $\varepsilon\mathsf{Q}$. The Dirichlet Laplacian is solved on A_ε. Solutions u_ε are associated with a right-hand side f that is real analytic on the closure of the unperturbed domain A_0.

The solution u_0 of the limit problem on A_0 has an infinite singular function expansion at the vertex $\{0\}$ that converges only conditionally, but can be rearranged so that it converges in a neighborhood of $\{0\}$, see [1]. The aim of the talk is to show how this result extends to the perforated domains A_ε.

The strategy relies on a combination of odd reflections and conformal mappings so that the original problem is transformed into a similar problem where the perforations are near the center of a disc. An integral representation [2] can then be used, paving the way for the Functional Analytic Approach [3,5]. The final outcome can be compared with the multi-scale expansions [4] for which convergence does not hold in general.

Based on joint work with Martin Costabel, Matteo Dalla Riva, and Paolo Musolino.

Keywords: Dirichlet problem, double layer potential, stable asymptotics.

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How can the blow up of the lunar perigee explain the instability of suspension bridges?

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In 1886, Hill introduced a class of linear second order ODE's with periodic coefficients, aiming to describe the lunar perigee; these equations generalize earlier equations introduced by Mathieu in 1868. In 1905, in his presentation of the mathematical works by Hill, Poincaré emphasizes that, among his discoveries, *...celle qui fera son nom immortel, c'est sa théorie de la Lune.* Poincaré was well aware that the Moon theory was not the only application of the Hill equation, his last comment on the equation reads *...quand elles s'étendront à un domaine plus vaste, on ne devra pas oublier que c'est à M. Hill que nous devons un instrument si précieux.* It is well-known that Poincaré was right, the very same equation is nowadays used to describe many different natural phenomena.

In this talk, we explain how the Hill equations can be used to study the instability of suspension bridges. We start with a simple finite system of ODE's of Mathieu-type and we end up with infinite dimensional dynamical systems such as nonlinear beam and/or plate equations. The resulting PDE's are of fourth order in space, reflecting the bending energy appearing in these models.

The instability of suspension bridges may lead to unexpected torsional oscillations which were the main cause of some historical collapses. But to detect instabilities in an infinite dimensional dynamical system is a very difficult task. We show that this problem may be solved through a finite dimensional approximation with accurate estimates of the errors.

Based on joint works with Elvise Berchio, Alberto Ferrero, Maurizio Garrione, Chiara Zanini.

Keywords: Hill equation, stability of modes, beam equations, suspension bridges.

The Technique of Hierarchical Matrices

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The technique of hierarchical matrices tries to approximate fully populated large-scale matrices in a representation only requiring a storage of $O(n \log^* n)$. Of similar importance is the fact that all matrix operations (including computing the inverse matrix and the LU decomposition) can be also performed with almost linear cost and high accuracy. Also the fully populated matrices of boundary elements methods can be treated in this way.

We address different methods how to generate the BEM matrix efficiently. Furthermore, we mention the variant of the $\mathcal{H}^2$ matrices. In the latter case, even a linear cost of $O(n)$ can be achieved.

Deeper theoretical considerations ensure that, in the elliptic case, the inverse of finite element stiffness matrices can be well approximated by hierarchical matrices. This statement does not require smoothness of the pde or the solution. A related subject is the (approximate) LU factorisation, which allows us to construct fast iterative methods in a blackbox fashion.

Having at hand all matrix operations, we can also compute matrix functions and solve matrix equations.

Keywords: matrix algorithms, boundary element methods, matrix equations, matrix functions.

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Multi-Trace Boundary Element Methods

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The scattering of acoustic or electromagnetic waves at a penetrable object composed of different homogeneous materials can be modelled by means of boundary integral equations (BIE) posed on the interfaces. This approach is widely used in numerical simulations and often relies on so-called first-kind single-trace BIE [1, Sect. 2]. Their boundary element Galerkin discretization gives rise to poorly conditioned linear systems, for which the otherwise successful operator (Calderón) preconditioning approach does not seem to be available.

As a remedy we propose new multi-trace boundary integral equations; whereas the single-trace BIE feature unique Cauchy traces on sub-domain interfaces as unknowns, the multi-trace idea takes the cue from domain decomposition and tears the unknowns apart so that *local* Cauchy traces are recovered. The benefit is the possibility of straightforward Calderón preconditioning.

Multi-trace formulations come in two flavors. A first variant, the *global multi-trace approach*, is obtained from the single-trace equations by taking a "vanishing gap limit" [1, Sect. 3]. The second variant is the *local multi-trace method* and is based on local coupling across sub-domain interfaces [1, Sect. 4]. Numerical experiments for acoustic scattering demonstrate the efficacy of Calderón preconditioning.

Based on joint work with Xavier Claeys (Laboratoire Jacques-Louis Lions, UPMC, Paris VI) and Carlos Jerez-Hanckes (Pontifica Universitad Catolica de Chile, Santiago de Chile).

Keywords: transmission problems, multi-trace boundary integral equations, PMCHWT, operator preconditioning, boundary elements.

Survey article

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Space time boundary element methods for the heat equation

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While the discretization of time dependent partial differential equations is usually done via explicit or implicit time stepping schemes, we propose to use a global approach in the space time domain. For the model problem of the heat equation we first summarize known results on the formulation of boundary integral equations and the mapping properties of all involved boundary integral operators. It turns out that almost all results follow as in the case of the Laplace operator. In fact, the numerical analysis of related boundary element methods can be done in the same favour. In particular this concerns the formulation of appropriate preconditioning strategies as well as of suitable a posteriori error estimators to drive an adaptive discretization scheme simultaneously in space and time. Numerical examples will be given which confirm the theoretical findings. In addition we will present first results for the space time boundary element discretization of the wave equation.

Based on joint work with S. Dohr, K. Niino, and M. Zank.

Keywords: heat equation, boundary integral equations, space time BEM.

Operator theory and applications: a fruitful interplay

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The theory of linear operators is an important tool to investigate the linear stability of physical systems or their time evolution. In turn, applications from physics and engineering have contributed considerably to the advances of operator theory. The most prominent example of this fruitful interaction is the interplay between quantum mechanics and the theory of self-adjoint operators in Hilbert spaces.

In this talk more recent applications of operator theory are presented, including such different areas as (magneto-)hydrodynamics and photonic crystals. Special attention is paid to eigenvalue problems where numerical calculations may fail to produce reliable results, such as non-selfadjoint problems; here rigorous analytical information provided by operator theoretic methods is highly desirable.

Keywords: Spectrum, perturbation theory, guaranteed eigenvalue enclosures, linear stability.

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SECTION 2

Conference Talks

An L^1-Product-Integration Method in Astrophysics

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We consider a weakly singular kernel Fredholm integral equation of the second kind in $L^1([a,b],\mathbb{C})$. Sufficient conditions are given for the existence and uniqueness of the solution. We extend the product-integration method from $C^0([a,b],\mathbb{C})$ to $L^1([a,b],\mathbb{C})$. To improve the accuracy of the approximate solution, we use three different iterative refinement schemes. An application is given in Astrophysics with the transfer stellar atmosphere equation.

Based on joint work with Hanane Kaboul.

Keywords: Fredholm integral equations, product-integration method, iterative refinement schemes.

Computational efficiency for Herberthson's integral equation

Benjamin ALZAIX

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In this presentation, the scattering of an electromagnetic plane wave incident on a perfectly conducting smooth surface is considered. We analyse the properties of an integral equation, introduced by M. Herberthson, which adapts the Electric Field Integral Equation (EFIE) to plane-wave scattering by means of a phase-conjugated pseudo-current. We write HEFIE for this modified EFIE. First, we break down the equation as the sum of the conventional EFIE and a perturbation which we prove to be compact. Then we focus on the solution of the HEFIE by a Galerkin method. Using previous work by S.H. Christiansen, the HEFIE is presented as a saddle point problem and we etablish optimal convergence estimates similar to those for the conventional EFIE. We show, via numerical experiments, that the phase conjugation in the HEFIE allows for a model order reduction which leads to increased computational efficiency at high frequencies.

Based on joint work with Bas Michielsen, Jean-René Poirier and Luc Giraud.

Keywords: Electromagnetic scattering, Boundary Integral Equation, Convergence estimates, Galerkin method.

Scalable and view-independent multi-projector display for arbitrary surface

Shahab ASKARIAN

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According to recent papers, most existing systems for calibrating multi-projector display suffered from several important limitations such as dependence on point of view, restriction on the display surface moreover number of projectors and using obtrusive markers. In this paper, we have proposed a new method to calibrate a multi-projector display system which removes all mentioned restrictions. We introduce camera-projector pair as a set which put together to project a unified image on arbitrary display surface. First of all, internal and external calibration of each set is done. After that, the calibration of multi-projector display system reduces to a camera pose estimation problem. According to the stereo vision system in each set and availability of 3D coordinate of feature points, camera pose estimation problem have been done by using a robust to noise geometric registration of 3D point algorithm. Next, we put the mentioned pairs around a 3D object with any size and extract 3D shape of object. Finally, the extracted shape is painted by using a graphical software. The painted shape is assumed as one frame of the final video. This process is repeated several times to produce the final video.

Based on joint work with H. R. Pourreza.

Keywords: 3D point registration, camera pose estimation, multi-projector display.

The SVD of the Poisson Kernel

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This talk will describe the singular value decomposition (SVD) of the Poisson kernel regarded as a compact linear map of $L^2(\partial\Omega)$ to the space $L^2_H(\Omega)$ of harmonic functions on a bounded region $\Omega \subset R^N$. In particular the singular values and the singular vectors will be described together with a different formula for the reproducing kernel of the space $L^2_H(\Omega)$. The results are used to find an explicit bound in an inequality of Hassell and Tao.

Keywords: Poisson kernel, harmonic Bergman space, SVD.

Analysis of Boundary-Domain Integral Equations For Variable Coefficient Mixed BVP in 2D

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In this paper, the mixed boundary value problem for the second order “stationary heat transfer” partial differential equation with variable coefficient is considered in 2D. Using an appropriate parametrix (Levi function), this problem is transformed to some direct segregated Boundary-Domain Integral Equations (BDIEs). Although the theory of BDIEs in 3D is well developed, the BDIEs in 2D need a special consideration due to their different equivalence properties. Consequently, we need to set conditions on the domain or on the associated Sobolev spaces to insure the invertibility of corresponding parametrix-based integral layer potentials and hence the unique sovability of BDIEs. The properties of corresponding potential operators are investigated. The equivalence of the original BVP and the obtained BDIEs are analysed and the invertibility of the BDIE operators is proved.

Based on joint work with T.T. Dufera and S.E. Mikhailov.

Keywords: Mixed Boundary Value Problem, Boundary-Domain Integral Equation, Equivalenc and Invertability.

On Fredholm integral equations

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Consider the integral equation:

$$Y_2\left(t\right) - \mu \int_a^b \Delta_{\mathbb{k}}\left(t,s\right) Y_1\left(s\right) ds = G\left(t\right) \tag{1}$$

where, $x : [a,b] \to \mathbb{R}$ is unknown, $G : [a,b] \to \mathbb{R}$ and $F_1, F_2 : \mathbb{R} \to \mathbb{R}$, are known and $Y_1Y_2 : [a,b] \to \mathbb{R}$ is such that $F_1 \circ x = Y_1$ $F_2 \circ x = Y_2$,, μ is a parameter. The kernel $\Delta_{\mathbb{k}}$of the integral equation is defined on $\Pi^2_{[a,b]} = \{(t,s) : a \leq t, s \leq b\}$. Equation (1) has been solved in the literature for the case $F_1 = F_2 = I$ (the identity mapping on $\mathbb{R}$) as Fredholm integral equation of 2$^{\text{nd}}$ kind. The purpose of this talk is to discuss the existence and uniqueness of solution of this equation for the case $F_1 \neq F_2 \neq I$ under various assumptions on the functions involved. A coincidence point theorem coupled with a function space $(C\left[a,b\right], \mathbb{R})$ and a contractive inequality are used to establish the result.

Keywords: coincidence point, contractive mapping, Fredholm type integral equation.

On the Convergence of Moving Average Processes for Dependent Random Variables

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Let $\{Y_i, -\infty < i < \infty\}$ be a doubly infinite sequence of identically distributed and ψ-mixing random variables with zero means and finite variance and $\{a_i, -\infty < i < \infty\}$ an absolutely summable sequence of real numbers. In this paper, we prove the complete moment convergence of $\{\sum_{k=1}^{n}\sum_{i=-\infty}^{\infty} a_{i+k}Y_i/n^{\frac{1}{p}}; n \geq 1\}$ under some suitable conditions.

Keywords: Complete moment convergence, Moving average, ψ-mixing.

Rocket exhaust dispersion model: deposition parameters analysis

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This work presents sensitivity studies regarding the deposition parameters of a two-dimension analytical model for rocket exhaust dispersion assessment. The model uses the Generalized Integral Laplace Transform Technique (GILTT) for computing advection and diffusion, and well documented schemes for deposition parameterization.
Simulations are made using meteorological data from Alcantara Launch Centre, Brazil, and different scenarios in order to evaluate the model response concerning both the planetary boundary layer (PBL) stability and the pollutant physical properties (physical estate and size). The chosen tracers are CO_2 (gaseous) and Al_2O_3 (particulate - with assumed diameters of 0.1, 2.5 and $10\mu m$), which are common products of rocket fuel combustion. Preliminary results show that there is no significative diference among the finer particulate and the gaseous tracer, for either stable and unstable PBL. For the coarser particulate, however, the deposition parameters, regardless of being of a relatively small contribution to dispersion, increase the ground level concentration in more than 8%.

Based on joint work with Daniela Buske and Regis S. De Quadros.

Keywords: GILTT, deposition, stability.

On the decomposition of time dependent reactor neutron spectra into fission, intermediate and thermal distributions: a new Monte Carlo solver with continuous energy

Luiz Felipe F.C. BARCELLOS

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In this contribution the time dependent continuous energy neutron spectrum is decomposed into three probability distributions (fission, intermediate and thermal). Two of the distributions preserve shape and the intermediate distribution is unknown and varies with time and is determined from Monte Carlo simulation. For the high and intermediate energy distribution the Monte Carlo simulation takes into account tracking and interaction of neutrons, whereas the thermal equilibrium regime is simulated considering only the reaction rates. Due to the equilibrium condition, energy gain and loss through scattering are balanced that turns tracking obsolete. The Monte Carlo implementation is then used to determine the transition rates between the three distributions, that after insertion in a differential equation system models the kinetics of the afore mentioned distributions. We solve the resulting equation system and determine the time evolution of the three solutions as well as their transient and stationary integral ratios.

Based on joint work with Bardo E.J. Bodmann, Sérgio Q. Bogado Leite, Marco T. de Vilhena.

Keywords: Neutronics, continuous energy, Monte Carlo.

Shifting strategy in the spectral analysis for the spectral Green's function nodal method for adjoint S_N problems in slab–geometry

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We present the positive features in the shifting strategy we use in the homogeneus component of the general solution of the monoenergetic, slab–geometry, adjoint discrete ordinates (S_N) equations inside each discretization node for neutral particle source–detector transport problems. The adjoint spectral Green's function (*SGF*) method uses the standard spatially discretized adjoint S_N balance equations and nonstandard *SGF* adjoint auxiliary equations, which have parameters that need be determined to preserve this local solution. We remark that the shifting strategy scales the N exponential functions of the local solution in the interval (0, 1]. One advantage is to avoid the overflow in computational finite arithmetic calculations

in high–order angular quadrature and/or coarse–mesh calculations. Another advantage is that the matrix, whose entries are the aforementioned parameters, becomes the same as the corresponding matrix for the forward S_N problem. We intend to use this strategy in X, Y geometry adjoint S_N problems.

Based on joint work with Odair P. da Silva, Carlos R. García Hernández, Jesús Pérez Curbelo.

Keywords: source–detector problems, adjoint discrete ordinates, shifting strategy.

The use of similarity indices in the analysis of temporal distribution of mammals

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Miriam BELMAKER

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Mammal community structure is comprised of multidimensional data. This paper analyzes the temporal distribution of mammals in the early Pleistocene site of ‘Ubeidiya, Israel. The statistical model reflects the relationship between communities from the same local but temporarily distinct, by means of a spatial correlation.

Resemblance matrices are computed for the response—that is, the dependent variable (in this case, the mammalian community structure)—and for the explanatory (independent) variables, for example, carnivore and human selectivity, by means of similarity indices. The connection between these matrices is computed through the application of the Mantel and Partially Mantel tests, as shown by the relevant model equations. The statistical significance is estimated with 10,000 permutation repetitions.

Keywords: similarity indices, matrix analysis, community structure, paleoanthropology.

A new diffeomorph conformal methodology to solve flow problems with complex boundaries by an equivalent plane parallel problem

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Bardo E.J. BODMANN

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The motivation for the present contribution are flow problems that may have boundaries with complex geometry. Conventional approaches use typically step wise approximations, that allow to cast the problem in schemes for established numerical solvers. The present approach is for continuous boundaries with some restrictions. More specifically, the problem with curvilinear boundaries is transformed in a problem with plane parallel boundaries by a diffeomorph conformal coordinate transformation. Consequently the operators of the dynamical equations change according to the additional terms from the affine connection. A case study that sketches the novel methodology together with a numerical simulation is presented.

Based on joint work with André Meneghetti, Marco T. de Vilhena.

Keywords: Flow problem, diffeomorph conformal transformation, equivalent plane parallel problem.

Generalized functions in the Calculus of Variations and in PDEs

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In the practice of calculus of variations and of partial differential equations, whenever a problem does not allow for a solution, suitable notions of generalized solutions are developed, usually by extending the class of admissible solutions. However, some problems allow for many generalized solutions, so that additional constraints are needed to single out the relevant solution. We introduce a new notion of solution by using functions of nonstandard analysis defined on a hyperfinite domain Λ satisfying $\mathbb{R} \subset \Lambda \subset {}^*\mathbb{R}$. These functions generalize the notion of distribution and of Young Measure, and in this setting the distributional derivative is represented by a finite difference operator with an infinitesimal step, thus allowing for a discrete nonstandard formulation of problems from the calculus of variations and from partial differential equations. By this formulation we are able to solve a wide range of problems, usually with a unique generalized solution. As an example, we will discuss the generalized solution of a variational problem that classically allows no solutions, and the generalized solution of an ill-posed PDE.

Keywords: nonstandard analysis, partial differential equations, calculus of variations.

MHD free convection in a nanofluid filled cavity with a circular body

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The present study focuses on the numerical investigation of the heat transfer enhancement of a Cu-water nanofluid in an enclosure involving a circular body under the effect of an inclined uniform magnetic field. The left and right walls of the cavity are kept at constant hot and cold temperatures, repectively, while the horizontal walls and the boundary of the circular body are assumed to be adiabatic. The coupled nonlinear equations of mass, momentum and energy governing the present problem are discretized using the dual reciprocity boundary element method which treats the nonlinear terms by the use of radial basis functions. Numerical simulations are carried out for a wide range of controlling parameters, such as Rayleigh number, Hartmann number, the solid volume fraction and the inclination angle of the magnetic field.

The results reveal that heat transfer and fluid flow are strongly affected by the presence of the circular body and the magnetic field.

Keywords: MHD free convection, nanofluid, dual reciprocity BEM.

Integer powers of certain complex $(2k+1)-$diagonal matrices

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Banded matrices have important role in applied mathematics. The applications of the banded matrices are seen in numerical analysis, differential and difference equations; especially, in the solution of boundary value problems, numerical of partial differential equations, delay differential equations. Also determinants, eigenvalues, eigenvectors and matrix powers of the tridiagonal, anti-tridiagonal, pentadiagonal, anti-pentadiagonal, circulant and skew-circulant matrices have many applications in the applied sciences. For example, Mazilu [I. Mazilu, D.A. Mazilu and H.T. Williams, Electronic Journal of Linear Algebra, 2012] used tridiagonal matrices to finding the magnetization dynamics of outline an approach.

We are studying a general expression for the entries of the lth power of $(2k+1)-$diagonal $n-$square matrix. Firstly, we give the eigenvalues and eigenvectors of $(2k+1)-$ diagonal $n-$ square matrix. Secondly, the $l-$th power of $(2k+1)-$ diagonal matrix we will get by using the expression $(A_k)^l = P_k(J_k)^l(P_k)^{-1}$, where J_k is the Jordan's form of A_k, P_k is the transforming matrix. Then, numerical examples are given. Finally, determinant of $(2k+1)-$ diagonal matrix obtain complex factorizations for Fibonacci polynomials.

Based on joint work with H. Kübra Duru.

Keywords: Chebyshev Polynomial, Power of Matrix, Fibonacci Polynomial.

Strategies for a parallel version to the SUPIM code

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National Institute for Space Research (INPE), Brazil

The operational ionosphere dynamics prediction system for the Brazilian Space Weather program is based on the Sheffield University Plasmasphere-Ionosphere Model (SUPIM). Several adaptations were done to become the SUPIM an operational forecasting code. A parallel version of the code was one improvement. Different strategies with focus on a parallel version to SUPIM code were considered: OpenMP, MPI, and many runs in parallel with the task manager OAR (a batch scheduler for large clusters). The development and the testing set for the parallel SUPIM are reported.

Based on joint work with Adriano Petry, Jonas R. Souza, André G. Pereira, Graham John Bailey

Keywords: Ionosphere, total electron content (TEC), parallel SUPIM.

Cancer dynamics: parameter analysis and identification

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Cancer is considered as a serious public health problem worldwide. There are some successful results against this disease, applying radiotherapy and chemotherapy. A mathematical model is formulated to describe the tumor dynamics and its interaction with chemotherapy drugs. The cancer dynamics is described by a nonlinear set of ordinary differential equations. The model embraces three different processes: cancer growth, regular cell dynamics, and angiogenesis. Each process is characterized by the associated parameters. A parameter sensitive analysis is performed. The parameter estimation is formulated as an inverse problem. Inverse solution is obtained by Differential Evolution (DE) meta-heuristics.

Based on joint work with Lídice Camps Echevarría, Helcio R. B. Orlande, Suani. T. R. Pinho, Orestes L. Santiago, Antonio J. Silva Neto.

Keywords: Cancer dynamics, parameter sensitivity analysis, inverse problem.

The method of superposition for near-field acoustic holography in a semi-anechoic chamber

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Near-field acoustic holography (NAH) is the process of reconstructing the vibrational behaviour of a structure from measurements of the acoustic field generated by these vibrations. Traditionally NAH was applied to planar regions where Fourier methods can be used to reconstruct the structural vibrations, even at frequencies beyond the sampling resolution limit. The Method of Superposition (MoS) is a relatively simple method for reconstructing the vibrational properties of structures with more general geometries. Recent work has shown that the MoS can also be effectively combined with sparse ℓ_1 regularisation to generate solutions from only a small number of terms in the superposition. In this work we discuss a reformulation of the MoS for NAH experiments in a semi-anechoic chamber; experiments in fully anechoic chambers can often prove impractical. In particular, we propose a modified Green's function approach for a semi-infinite domain with a hard reflecting boundary using the Method of Images and present the results of some supporting numerical experiments.

Based on joint work with Nadia Abusag.

Keywords: Method of Superposition, Near-field Acoustic Holography, Method of Images.

Dynamical Interface Crack Problems for Metallic and Electro-Magneto Elastic Composite Structures

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We consider 3-dimensional dynamical interface crack problems when the metallic and electro-magneto-elastic bodies are bonded along some proper parts of their boundaries where interface cracks occur. Using the Laplace transform, potential theory and theory of pseudodifferential equations on a manifold with boundary, we investigate the solvability and asymptotic properties of solutions to the crack problems under consideration. We prove the existence and uniqueness theorems and analyse the regularity and asymptotic properties of the mechanical and electro-magnetic fields near the crack edges and near the curves where the different boundary conditions collide. In particular, we characterize the stress singularity exponents and show that they can be explicitly calculated with the help of the principal homogeneous symbol matrices of the corresponding pseudodifferential operators. For some important classes of anisotropic media we derive explicit expressions for the corresponding stress singularity exponents and show that they essentially depend on the material parameters.The questions related to the so called oscillating singularities are treated in detail as well.

Based on joint work with T.Buchukuri and D.Natroshvili.

Keywords: Interface crack, 3-dimensional dynamical problems, asymptotic analysis.

Bending of Elastic Plates:Generalized Fourier Series Method

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A convenient way to approximate the solutions of boundary value problems for this mathematical model is to expand them in a complete set of functions that are intrinsically tied to the structure of the appropriate layer potentials. Computation based on such expansions, however, is hampered by the fact that the Gram-Schmidt process used to orthonormalize the set is numerically unstable. This paper aims to indicate a procedure that circumvents this obstacle and leads to good numerical results to within a prescribed accuracy. The technique is illustrated in application to the interior Dirichlet problem.

Based on joint work with Dale Doty.

Keywords: plates, bending, series.

An ADMM-AHM integrated approach for EDPs with rapidly oscillating coefficients

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Camila Pinto da COSTA

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The Advection-Diffusion Multilayer Method (ADMM) emerged to address the solution of advection-diffusion equations with variable coefficients in the context of pollutant dispersion modeling. The ADMM is based on the piecewise-constant approximation of the variable coefficients and the application of the Laplace transform. Applications of ADMM in other areas are potentially relevant for modeling the behavior of heterogeneous media. However, if such heterogeneity is characterized by rapidly oscillating coefficients, the direct application of the ADMM would increase the computational effort needed, as it would require a very fine discretization of the domain. In order to overcome such a drawback, in this contribution, an alternative approach combining the ADMM with the Asymptotic Homogenization Method (AHM) is presented. The ADMM-AHM integrated approach is compared to the direct application of the ADMM in order to assess the accuracy of the estimations of the solution of the original problem, and the computational efficiency.

Based on joint work with L. D. Pérez-Fernández and J. Bravo-Castillero.

Keywords: EDPs with rapidly oscillating coefficients, ADMM, AHM.

A level-set method for the computation of the Dirichlet-to-Neumann map

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Julien DAMBRINE

Université de Poitiers In this talk we focus on the computation of the Dirichlet-to-Neumann

operator for the Laplace equation, using a hypersingular boundary integral equation (HS-BIE). We propose a level-set based representation of the boundary, which, albeit not natural for boundary integral equations, is particularly convenient for problems involving a moving boundary such as cell motion, shape optimisation or water-waves. In a previous work of Kublik *et. al.*, such a method was used to solve a boundary integral problem, but the focus was on recovering the "bulk" solution. In this work we propose to extend this approach in order to solve the Dirichlet-to-Neumann problem.

Based on joint work with Nicolas Meunier.

Keywords: Dirichlet-to-Neumann, level-set, HSBIE

On one nonlocal contact problem for poisson's equation in 2d area

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The present work is devoted to the specific nonlocal statement and analysis of one contact problem for Poisson's equation in two-dimensional domain. For numerical solution the iteration process is constructed, which allows one to reduce the solution of the initial problem to the solution of a sequence of classical Dirichlet problems. The algorithm is suitable for parallel realization. The specific problem is considered as example and solved numerically.

Based on joint work with Hamlet Meladze.

Keywords: PDE, Nonlocal contact problem, Iteration process.

A dynamical based on meta-heuristic algorithm for optimization of airline crew pairing problem

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Airline operational problems associated with flight scheduling, aircraft scheduling and crew scheduling are usually solved in two phases: crew pairing and crew rostering and management of irregular operations. Each problem has its own complexities. This study examines the crew pairing problem, one of the most comprehensible problems encountered in airline planning. The study aims to generate a set of crew pairings with minimal cost, covering all flight legs and fulfilling legal criteria. Drawing from the previous studies which seek to solve the crew pairing problem through genetic algorithm, the present study proposes a multi-objective genetic algorithm approach. To test the algorithm, optimal crew pairings have been generated by making use of the flight data obtained from an airline company operating in Turkey.

Based on joint work with Nihan Cetin Demirel.

Keywords: airline crew pairing, set-covering problem, genetic algorithm.

Spectral Boundary Element Algorithms for Multi-Length Interfacial Dynamics

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Interfacial dynamics in Stokes flow via the solution of boundary integral equations has developed considerably in the last decades. The main benefits of this approach are the reduction of the problem dimensionality by one and the great parallel scalability. In this talk we will present our efforts to study efficiently multi-length interfacial dynamics in Stokes flow, such as the drop coalescence process, droplets and cells in close proximity to microchannel walls as well as tips and necks during large interfacial deformations.

For the accurate determination of these challenging three-dimensional problems, we have developed a series of efficient and highly-accurate interfacial algorithms based on our Spectral Boundary Element implementation for Stokes flow. As applications for multi-length interfacial systems, we will present our investigation of large deformation of soft particles, involving pointed tips and tails in micro-channels. We will also present our investigation of dye particles which can be orders of magnitude smaller than the channel size.

Keywords: boundary integral methods, spectral methods, stokes flow.

Effective elastic properties of random two - dimensional composites

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Consider 2D two-phase random composites with circular inclusions of concentration f. New analytical formulae for the effective constants are deduced up to $O(f^4)$ for macroscopically isotropic composites. It is shown that the second order terms $O(f^2)$ do not depend on the location of inclusions whilst the third order terms do. This implies that the previous analytical methods (effective medium approximation, differential scheme, Mori-Tanaka approach and so forth) can be valid at most up to $O(f^3)$ for macroscopically isotropic composites. First, the local elastic field for a finite number n of inclusions arbitrarily located on the plane are found by a method of functional equations. Further, the limit $n \to \infty$ yields conditionally convergent series defined by the Eisenstein summation method. One of the series for periodic composites is the famous lattice sum $S_2 = \pi$ deduced by Rayleigh for a conductivity problem.

Based on joint work with Vladimir Mityushev.

Keywords: effective elastic moduli, complex potentials, random composites.

Mixed boundary value problems for the Helmholtz equation in a model 2D angular domain

Roland DUDUCHAVA

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A model mixed boundary value problems for the Helmholtz equation in a planar angular domain is investigated, when a Dirichlet data is prescribed on one side and a Neumann data-on another one. The BVP is considered in a non-classical setting when solutions are sought in the Bessel potential spaces H^s_p, $1 < p < \infty$, $s > 1/p$. The problem is investigated using the potential method by reducing them to an equivalent boundary integral equation (BIE) in the Sobolev-Slobodeckij space on a semi-infinite axes. The obtained BIE is of Mellin convolution type and are investigated by applying the recent results on Mellin convolution equations in Bessel potential spaces obtained by V. Didenko & R. Duduchava.

Explicit conditions of the unique solvability of this BIE are found and used to write explicit conditions for the Fredhoilm property and unique solvability of the initial model BVPs for the Helmholtz equation in the above mentioned non-classical setting.

Based on joint work with Medea Tsaava.

Keywords: Helmholtz equation, 2D angle, Mellin convolution, Bessel potential space.

Analysis of Boundary-Domain Integral Equations For Neumann BVP with Variable Coefficient in 2D

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In this paper, the Neumann boundary value problem for the second order “stationary heat transfer” elliptic partial differential equation with variable coefficient is considered in 2D. Using an appropriate parametrix (Levi function), this problem is transformed to some direct segregated Boundary-Domain Integral Equations (BDIEs). Although the theory of BDIEs in 3D is well developed, the BDIEs in 2D need a special consideration due to their different equivalence properties. Consequently, we need to set conditions on the domain or on the associated Sobolev spaces to insure the invertibility of corresponding parametrix-based integral layer potentials and hence the unique sovability of BDIEs. The properties of corresponding potential operators are investigated. The equivalence of the BDIEs to the original BVP, BDIEs solvability, solution uniqueness/non uniqueness, as well as Fredholm property and invertibility of the BDIEs operator are analyzed. It is shown that the BDIE operators for Neumann BVP

in a bounded domain are not invertible, and appropriate finite-dimensional perturbations are constructed leading to invertibility of the perturbed operators.

Based on joint work with S.E Mikhailov and T.G Ayele.

Keywords: Neumann Boundary value problem, Boundary-Domain Integral Equation, Equivalence and Invertability.

On the neutron point kinetic equation with reactivity decomposition based on two time scales

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In the present contribution we present a neutron kinetic equation, where the reactivity is decomposed in terms with respect to two time scales. The short one may be associated to the traditional point kinetic model, whereas the large time scale is due to some properties related to burn-up, i.e. effects due to neutron poisons. The resulting equation system is a non-linear one so that one has to resort to a recursive system, where the homogeneous equation of the recursive system is linear and the non-linear contributions are present as known source term from previous recursion steps. We present case studies with severe time structure in order to show the robustness of the present approach for this kind of problem.

Based on joint work with Bardo E.J. Bodmann.

Keywords: Neutron diffusion, decomposition method, non-linear couplings.

On the solution of the multi-group neutron transport equation with error estimates

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Analytical solutions for mono-energetic neutron transport equation is well documented in the literature. Recently, the solution of the multi-group problem was solved in a hierarchical fashion, based on the solution of the mono-energetic problem. To this end a recursive system of mono-energetic transport equations is constructed, where the scattering terms from and into specific energy groups are considered as source. The recursive scheme is set-up such that the recursion initialisation satisfies the original boundary and initial conditions, whereas the remaining recursion steps are subject to homogeneous conditions. The recursion depth is related to the prescribed accuracy. Numerical results including error estimates are reported for a problem with pulsed neutron source.

Based on joint work with Bardo E.J. Bodmann, Marco T. de Vilhena.

Keywords: Neutron transport, multi-group solution, heuristic convergence.

Iterated Kantorovich versus Kulkarni method for Fredholm integral equations

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From all standard projection approximations of a bounded linear operator in a Banach or Hilbert space, iterated Kantorovich and Kulkarni's discretization exhibit a global superconvergent error bound.
We compare these discretizations, both from the superconvergence point of view and their computational cost, when applied to a Fredholm integral equation of the second kind.

Based on joint work with Filomena D. d'Almeida.

Keywords: projection approximations, integral operators, error bounds.

Fundamental solutions of multi-dimensional time - fractional operators

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We present explicit formulas for the fundamental solution of the multi-dimensional time fractional diffusion-wave operator and time fractional parabolic Dirac operator. In these operators we replace the first-order time derivative by the Caputo fractional derivative of order $\beta \in]0,2]$. Applying operational techniques via the multi-dimensional Fourier transform and the Mellin transform we write the fundamental solution of the time fractional diffusion-wave operator as a Mellin-Barnes integral. This integral is then evaluated using the Residue Theorem and taking into account the parity of the dimension. The fundamental solution can be rewritten using Wright and Fox-Wright functions. From the obtained fundamental solution we deduce the fundamental solution of the time fractional parabolic Dirac operator, which factorizes the time-fractional diffusion-wave operator. To illustrate our results we present some plots of the fundamental solution for some particular values of the fractional parameter and the dimension.

Based on joint work with N. Vieira (CIDMA, University of Aveiro, Portugal).

Keywords: Time-fractional diffusion-wave operator, Time-fractional parabolic Dirac operator, Fundamental solutions.

Resonances for bubbly media

Brian FITZPATRICK

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Bubbles in medical ultrasonic imaging serve as strong sound scatterers at particular frequencies known as Minnaert resonances. At the Minnaert resonance it is possible to achieve superfocusing of acoustic waves or imaging of passive sources with a resolution beyond the Rayleigh diffraction limit. This talk is dedicated to the rigorous derivation of the Minneart resonance for arbitrarily shaped bubbles. It covers the following: (i) The application of layer potential techniques and Gohberg-Sigal theory to derive formulas for the Minnaert resonance of a single bubble in 2D and 3D; (ii) a Mathematical justification for the monopole approximation of scattering of acoustic waves by bubbles at their Minnaert resonant frequency; (iii) numerical validation of the 2D formula and numerical calculation of the Minnaert resonances for a 2 bubble system.

Based on joint work with Habib Ammari (ETHZ, Switzerland), David Gontier (ETHZ, Switzerland), Hyundae Lee (INHA, Korea), and Hai Zhang (UST, Hong Kong).

Keywords: Minnaert resonance, layer potentials, acoustic waves.

A New Family of Boundary Domain Integral Equations for a Mixed Elliptic BVP with Variable Coefficient in Unbounded Domains

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A mixed boundary value problem for the stationary heat transfer partial differential equation with variable coefficient is reduced to a system of direct segregated parametrix-based Boundary-Domain Integral Equations (BDIEs). We use a parametrix different from the one employed by Chkadua, Mikhailov, Natroshvili in *Analysis of direct boundary-domain integral equations for a mixed BVP with variable coefficient I: Equivalence and Invertibility* (2009). Mapping properties of the potential type integral operators appearing in these equations are presented in appropriate Sobolev spaces. We prove the equivalence between the original BVP and the corresponding BDIE system. The invertibility and Fredholm properties of the boundary-domain integral operators are also analysed. Analogous results for bounded domains are also obtained in *A New Family o Boundary-Domain Integral Equations for a Mixed Elliptic BVP with Variable Coefficient*(2015) by Fresneda-Portillo and Mikhailov.

Based on joint work with S. Mikhailov.

Keywords: Boundary-domain integral equations, variable coefficient, exterior problem.

Simulation in porous media: a functional solution for unsaturated media

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Based on a methodology to construct a parametrised solution for the Richards equation in terms of a functional solution, we show the universality of this solution for physically meaningful parameter sets. The matrix potential profile is evaluated by a self-consistency test showing only small differences between the true and the parametrised solution. In this contribution we continue to explore the validity and limitations of this solution and determine a connection between the formal solution parameter set and the physical parameter set. In this work we analyse the sensitivity of the solution in relation to variation of each of these parameters and the capability to represent solutions for different physically meaningful scenarios. The numerical results are performed by simulations using average values of soil parameters and water for a collection of twelve kinds of different soil textures and compare our findings to numerical results by the SWAP software.

Based on joint work with with Bardo E.J. Bodmann, Marco T. de Vilhena.

Keywords: Richards equation, porous media, functional solution.

Numerical solution of nonlinear mixed Volterra - Fredholm integral equations in complex plane via PQWs

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This paper presents an efficient numerical method for solving nonlinear mixed Volterra- integral equations in complex plane. The method is based on fixed point iteration procedure. In each iteration of this method, periodic quasi-wavelets are used as basis functions to approximate the solution. Also, by using Banach fixed point theorem, some results concerning the error analysis are obtained. Finally, some numerical examples show the implementation and accuracy of this method.

Based on joint work with Hossein Beiglo.

Keywords: Nonlinear mixed Volterra-Fredholm integral equations, Periodic quasi-wavelet, Complex plane, Fixed point theorem.

Automatic separation of distinct retinal vessels

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The separation of retinal vessels is an important part of any automatic system for detection of vessel abnormalities caused by several systemic diseases such as diabetes, hyper tension and other cardiovascular conditions. It also provides a platform for medical researches on relation of various diseases and vessel characteristics. This paper presents a graph-based method for automatic separation of retinal vessels. In the proposed method, we first model the vessel structure as a directed graph where each node represents an intersection point in the vascular tree, and each link corresponds to a vessel segment between two intersection points. Link directions are based on the vessel growth direction in the retina. In the next step, the extracted graph is analyzed to find the most acceptable label for each link. The result is compared with the manually labeled images of the DRIVE dataset and the accuracy value of 65% is obtained.

Based on joint work with Hamidreza Pourreza.

Keywords: Retinal Vessel, Separation, Graph.

Solvability of some inverse problems for kinetic equations

Fikret GOLGELEYEN

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We consider uniqueness, existence and stability of solution of some inverse problems for kinetic equations. Morover, we propose some numerical algorithms to obtain the approximate solution of the problem.

Based on joint work with I. Golgeleyen.

Keywords: kinetic equation, inverse problem, solvability.

On the solution of some inverse and ill-posed problems for evolution equations and functional equations

Ismet GOLGELEYEN

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In this work, we investigate the solution of some inverse and ill-posed problems for evolution equations. By using the given data we reduce the problem to a functional equation. Then we use the tools of the theory of functional equations to obtain the solution.

Keywords: Evolution equation, inverse problem, functional equation.

A Mathematical Model Of Cell Clustering Due To Chemotaxis

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In biological experiments small clusters of cells have been observed to move together and combine to form larger clusters of cells. These cells move by a process called chemotaxis where the cells detect a chemical signal and its gradient, and move in the direction in which the signal is increasing.

A number of mathematical models for simulating the motion of cells due to chemotaxis have been proposed, ranging from simple diffusion-reaction equations for finding the density of the cells to complete simulations of how the chemical receptors on the cell membrane react to the chemical signal and cause the cell membrane to move. This work presents a simple equations of motion model to describe how the cells move which is coupled to a diffusion equation solution of how the chemical signal spreads out from individual cells.

The talk will be illustrated with some typical examples.

Keywords: Chemotaxis, Cell Clustering, Mathematical Modeling.

An acceleration approach for fracture problems in the extended boundary element method (XBEM) framework

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The boundary element method (BEM) is more accurate and stable for fracture mechanics problems than the more popular domain discretisation methods such as the finite element method (FEM). However, the displacement and stress fields around the crack tip have to be represented adequately in order to obtain a meaningful solution of the problem. Some authors have used different approaches to tackle this issue in BEM, one can cite the quarter-point and the partition of unity. More recently, the authors have developed an enriched formulation for the BEM (XBEM) in 2D anisotropic fracture problems, where enrichment functions are embedded into the BEM formulation. Nevertheless, the final linear system of equations $\mathbf{Ax} = \mathbf{b}$ is fully populated, thus requiring large computational time to solve. Here we introduce the adaptive cross approximation (ACA) as an acceleration technique, in order to reduce the solution time for XBEM formulations. The ACA uses low-rank approximation combined with hierarchical matrices to reduce the number of terms of the $\mathbf{A}$ matrix which have to be calculated. This combined approach using ACA and XBEM is presented for the first time for 2D anisotropic problems.

Based on joint work with Sam Kettle, Jon Trevelyan, Lucas Campos and Éder Albuquerque.

Keywords: Anisotropic materials, extended boundary element method, adaptive cross approximation.

Kinetic Modeling of Carbon-11 Pitsburgh Compound-B ($[^{11}C]PIB$) with Laplace Transform

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The Carbon-11 Pitsburgh Compound-B ($[^{11}C]PIB$) is a radiopharmaceutical that allows the in vivo visualization of Aβ plaques in the brain with positron emission tomography(PET) imaging . We propose a two-tissue reversible compartmental model as the mathematical modeling in the quantitative analysis of $[^{11}C]PIB$, to improve the diagnosis of diseases, such as Alzheimer's Disease (AD). We apply Laplace Transform technique to solve a system of two first order differential equations corresponding to the two compartments. We calculate the image-derived input function assuming certain relations between the kinetics of the region of interest(Cortex) and the refe-

rence region(Cerebellum) and their parameters are computed by nonlinear regressions. We apply our results to quantify Aβ plaques in the Cortex of a three month-old swiss mice using a $[^{11}C]PIB$ μPET scan. In summary, more refined kinetic modeling of [11C]PIB μPET offers important opportunities to advance in the understanding and diagnosis of AD.

Based on joint work with Evandro Manica, Gianina T. Venturin , Samuel Greggio, Eduardo R. Zimmer, Jaderson C. da Costa.

Keywords: Kinetic Modeling, Laplace Transform, Alzheimer Disease.

Flux Characterization in Heterogeneous Transport Problems by the Boundary Integral Method

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Randy HAZLETT

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Potential flow problems with spatially varying transport coefficients are commonplace. While numerical methods are often implemented for such problems, coupled analytic solutions to heterogeneous, anisotropic problems enable below-grid, nonlinear flow effects to be accurately captured, even for systems with complex source functions. Coupling occurs through construction of boundary integrals to capture material transport between homogeneous domains. Various nonparametric methods are examined to compute the unknown flux distribution, including Gauss node distribution schemes with point source functions and patch-wise, semi-analytic integration. The merits of a posed parametric method that asserts a hybrid boundary condition and avoids numerical integration altogether within an overall analytic structure are further examined here. Reduction of heterogeneous problems to equivalent homogeneous transport property problems yields another term with linkage to uniform flux and uniform pressure contributions at increasingly distance interfaces in prolonged systems. This is a critical step towards analytic simulation of more complex phenomena.

Keywords: BIM, heterogeneity, transport.

GPU based mixed precision PWR depletion calculation

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The correct and faster evaluation of fuel burnup is fundamental to achieve reliable results in refueling optimization. Recently, we have described a parallel GPU implementation of PWR reactor burnup using Runge-Kutta-Fehlberg method.

Depletion evaluation requires the solution of a system of first order, ordinary, coupled differential equations (ODE) which represent actinides and fission yields evolved in radioactive reaction chains. The depletion operator of PWR reactor core is represented by a large sparse matrix with more than one hundred thousand lines, that can be calculated by exponential matrix methods approach, as Taylor series expansion.

The implementation and results of both mixed-precision semi-implicit Adams-Moulton-Bashforth and Taylor expansion series methods in GPU were described in this study. These advance obtained to solve the fuel burnup in GPU reduces the computational effort to a fraction of time spent in mainstream code.

Based on joint work with F.C. Silva, A.S. Martinez.

Keywords: GPU, Burnup, Adams-Moulton-Bashforth.

Numerical Solution of Partial Differential Equations by the Taylor Series Method

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The C++ function written by arithmetic operations, functions, and conditional statements, etc. can be easily Taylor expansion.

As we can calculate the solution to any degree, we can be used as the higher order formula to replace the Runge-Kutta method. Using the Taylor series, error evaluation also easily be carried out, it can be easily obtained the appropriate step size within tolerance. The partial differential equations spatially differencing, we propose to solve by applying the series method to the simultaneous ordinary differential equations in the time direction which is obtained. This method can be highly stable and accurate calculations when you use it.

Keywords: Numerical Taylor series Method, Higher order, Higher accuracy.

Two dimensional Gauss-Hermite quadrature method for jump-diffusion partial-integro differential option pricing models

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Two asset jump-diffusion option pricing problem is modelled by a partial-integro differential equation (PIDE) depending on time and on two spatial variables. In this work a suitable change of variables is proposed to remove the cross derivative term. Finite difference method is used to solve the differential part of the PIDE transformed problem while the integral part

is approximated using a two dimensional Gauss-Hermite quadrature. The method results stable and consistent, providing reliable positive solutions. Numerical examples illustrate the properties of the solutions.

Based on joint work with M. Fakharany and R. Company.

Keywords: Numerical Integration, Gauss-Hermite Quadrature, Partial integro-differential equations.

Online Traffic Prediction Using Time Series: A Case Study

Ali KARIMPOUR

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The increase in the number of vehicles on the streets have made the traffic management a challenging issue. Therefore, it is necessary to predict the number of cars in the intersections online. To apply Intelligent Transportation Systems effectively, it is necessary to get access to the online data of traffic flow. In this paper, a time series model is proposed to predict the traffic flow for a certain intersection, which can also be utilized in other intersections. One of the advantages of a model based on time series compared to other models is that having previous traffic data of an intersection, it is possible for engineers to predict the upcoming traffic condition of that intersection. Implementing this method on real data demonstrated that the model is able to predict the traffic flow with 88.74% accuracy for 15 minutes ahead, 81.96% for 60, and 79.37% for 120 minutes ahead, respectively.

Based on joint work with Abolfazl Karimpour, Mostafa Karimpour, Kinoush Kompany.

Keywords: Time Series, Intelligent Transportation Systems, Traffic Flow Prediction.

C^*-algebras of Bergman type operators over domains with angles

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Given $\alpha \in (0, 2]$, we study the C^*-algebra $\mathfrak{A}_{\mathbb{K}_\alpha}$ generated by the operators of multiplication by piecewise constant functions with discontinuities on a finite union $\mathfrak{L}_\omega$ of rays starting from the origin and by the Bergman and anti-Bergman projections acting on the Lebesgue space $L^2(\mathbb{K}_\alpha)$ over the open sector

$$\mathbb{K}_\alpha = \{z = re^{i\theta} : r > 0,\ \theta \in (0, \pi\alpha)\}.$$

Then, for any simply connected domain U with $\infty \notin U$ and piecewise Dini-smooth boundary admitting non-zero inner angles, the C^*-algebra $\mathfrak{B}_U$ generated by the operators of multiplication by piecewise continuous functions with discontinuities on a finite union $\mathfrak{L} \subset U$ of non-tangent smooth arcs and by the Bergman and anti-Bergman projections acting on the Lebesgue space $L^2(U)$ is investigated. Symbol calculi for the C^*-algebras $\mathfrak{A}_{\mathbb{K}_\alpha}$ and $\mathfrak{B}_U$ are constructed and an invertibility criterion for the operators $A \in \mathfrak{A}_{\mathbb{K}_\alpha}$ and a Fredholm criterion for the operators $A \in \mathfrak{B}_U$ in terms of their symbols are established.

Based on joint work with Enrique Espinoza-Loyola.

Keywords: Bergman operator, polygonal domain, C^*-algebra.

The interior transmission eigenvalue problem for an inhomogeneous media with a conductive boundary

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Andreas KLEEFELD

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In this talk, the interior transmission eigenvalue problem for an inhomogeneous media with a conductive boundary condition is investigated. Discreteness and existence of the interior transmission eigenvalues is shown. The inverse spectral problem of gaining information about the material properties from the interior transmission eigenvalues is illustrated. In particular, it is proven that the first interior transmission eigenvalue is a monotonic function of the refractive index and the boundary conductivity parameter, and a uniqueness result for constant coefficients is obtained. Additionally, numerical examples in three dimensions are presented to demonstrate the theoretical results.

Based on joint work with O. Bondarenko and I. Harris.

Keywords: interior transmission eigenvalue, inhomogeneous media, conductive boundary condition.

Loewner theory in the study of univalent mappings in higher dimensions

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Gabriela KOHR

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In this talk we survey classical and also modern results in the theory of Loewner chains and the generalized Loewner differential equation on the unit ball in $\mathbb{C}^n$. Finally, we point out certain open problems and conjectures related to Loewner chains, Herglotz vector fields, and the generalized Loewner differential equation in $\mathbb{C}^n$.

Based on joint work with Ian Graham (Toronto) and Hidetaka Hamada (Fukuoka).

Keywords: Loewner PDE, Herglotz vector field, evolution family.

Boundary value problems for nonlinear Brinkman systems in Lipschitz domains

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Mirela KOHR

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In this talk we present recent existence and uniqueness results in Sobolev and Besov spaces for boundary value problems involving nonlinear Brinkman and Navier-Stokes systems with constant/variable coefficients in Lipschitz domains in Euclidean setting or in compact Riemannian manifolds. We use a layer potential method combined with a fixed point theorem to show the desired existence and uniqueness results.

Based on joint work with Massimo Lanza de Cristoforis (Padova), Sergey E. Mikhailov (London) and Wolfgang L. Wendland (Stuttgart).

Keywords: Nonlinear Brinkman system, layer potential operators, fixed point theorem.

On The Second Order Nonlinear q-Difference Equations With A Delay term

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Aydin KURNAZ

Department of Mathematics, Selcuk University

In this talk, second order nonlinear q-difference equations with a delay term in the space are studied. The problem is expressed as an integral equation and then the existence and uniqueness of the solutions of the problem is analysed.

Based on joint work with A.B.Koc and K. Cakirtas.

Keywords: difference equation, q-calculus.

On the radiative conductive transfer equation: A heuristic convergence criterion by stability analysis

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Recently, the radiative conductive transfer equation in cylinder geometry was solved in semi-analytical fashion by the collocation method in both angular variables, using the S_N procedure. Upon application of the decomposition method the resulting recursive system of S_N radiative transfer equations were evaluated by the Laplace transform technique considering the non-linear term as source. In the present work we prove a heuristic convergence of the discussed solution inspired by stability analysis criteria and taking into account the influence of the parameter sets. Finally, we report on some case studies with numerical results for the solutions and convergence behaviour.

Based on joint work with Bardo E.J. Bodmann, Júlio C.L. Fernandes, Marco Túllio de Vilhena.

Keywords: Radiative conductive transfer equation, cylinder geometry, semi-analytical solution.

Optimized random forest classification accuracy by private evaporative cooling feature selection

Trang LE

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Classification of individuals into disease categories from biological data is an important application of statistical learning in biomedical research. Feature selection methods are incorporated into classifier training to avoid false positive predictions caused by overfitting in very high-dimensional biological data. Recently, in domains outside of bioinformatics with large sample sizes, methods have been proposed to avoid overfitting based on differential privacy, such as differentially private random forest algorithm (Singh, ICACCI, 2014) and a framework for reusable holdout sets (Dwork, Science, 2015). We introduce a stochastic privacy-preserving feature selection algorithm to optimize random forest classification based on an analogy with the evaporative cooling of an atomic gas. The evaporation represents the backwards elimination of features and the temperature represents the privacy threshold. Using simulated data, we compare test accuracies of evaporative cooling selection with differential privacy, standard random forest, and thresholdout random forest.

Based on joint work with Dr. Brett McKinney.

Keywords: privacy-preserving, evporative cooling, random forest

Differential operators associated with Markov operators

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Let T be a Markov operator on $C(K)$, K being a convex compact subset of some locally convex Hausdorff space. L_T will be a natural linear operator associated with it that, if K is finite dimensional, is indeed a second order elliptic differential operator which degenerates on a suitable subset of K containing its extreme points.

We shall see that, under suitable assumptions on T, L_T is closable and its closure generates a Markov semigroup $(S(t))_{t\geq 0}$ on $C(K)$. This fact implies that the (abstract) Cauchy problem

$$\begin{cases} u'(t) = L_T(u)(t) & t > 0 \\ u(0) = u_0 & u_0 \in D(L_T) \end{cases} \tag{ACP}$$

admits a unique solution $u(t) = S(t)u_0$ $(t \geq 0)$. Since the operators L_T are related to the so-called Bernstein-Schnabl operators B_n associated with T via an asymptotic formula, we are able to approximate the semigroup, and hence the solution to (ACP), by means of B_n's. This approximation formula which allows us to transfer several preservation properties of B_n's to the semigroup by stressing, as a consequence, some spatial regularity properties for the solutions of the relevant initial-boundary value problem (ACP). Some concrete examples will be illustrated.

Based on joint work with F. Altomare, M. Cappelletti Montano and I. Raşa.

Keywords: Markov operator, Markov semigroup, Approximation of semigroup.

Double Layer Kernels on C^1 Domains and C^1 Curvilinear Polygons

Jeff E. LEWIS

University of Illinois at Chicago

The Multilinear Commutator Theorem of Alberto Calderón, Calixto Calderón, Eugene Fabes, Max Jodeit, and Nestor Rivière can be used to show that an *Analytic Double Layer Kernel* (ADLK) restricted to the boundary of a C^1 domain Ω fits into the symbolic calculus developed by Lewis, Renata Selvaggi and Irene Sisto for singular integral operators (S.I.O.) with C^0 coefficients on C^1 manifolds. In particular a system of S.I.O. is Fredholm on $L^p\,(\partial\Omega)$ if and only the symbol is a nonsingular homomorphism. If the boundary of the domain is a C^1 curvilinear polygon, the symbol of the restricted ADLK is constructed using the Mellin transform and can be extended to handle power weights near the vertices. Examples of *Analytic Double Layer Kernels* include the gradients of the fundamental solutions of the Lamé system of elasticity and the Stokes system of hydrostatics.

Keywords: double layer kernels, C^1 domains, curvilinear polygons.

Integral representations for solutions of some BVPs in elasticity theory

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Angelica MALASPINA

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In this talk we describe an indirect method (introduced for the first time by A. Cialdea for n-dimensional Laplacian) to solve the Dirichlet problem for Lamé system in a multiply connected domain of $\mathbb{R}^n$, $n \geq 2$. In particular we show how to represent the solution in terms of a simple layer potential, instead of the classical double layer potential. By using the same method we are able to find necessary and sufficient conditions under which the solution of the traction problem in a multiply connected domain can be represented by a double layer potential. In our approach neither the knowledge of pseudo differential operators nor the use of hypersingular integrals are required. The method hinges on the theory of singular integral operators and on the theory of differential forms.

Based on joint work with A. Cialdea and V. Leonessa.

Keywords: Lamé system, potential theory, boundary integral equations.

Determination of surface soil temperature using a numerical solution to heat conduction equation

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Silvana MALDANER

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A numerical solution to heat conduction equation is proposed to determine the surface soil temperature. Experimental data of solar radiation, collected in southern Brazil, are employed in the numerical solutions. To evaluate the new model experimental, data of surface soil temperature are used. These data were collected at a depth of 5 cm. Soil temperature estimates from the heat conduction equation show reasonable agreement with the observed temperature time series.

Based on joint work with Gervasio Annes Degrazia, Virnei da Silva Moreira, Lidiane Buligon, Daniele Aimi, Cecilia P. Ferreira, Michel Stefanello, Debora Regina Roberti, Hans Rogerio Zimermann, Umberto Rizza .

Keywords: surface soil temperature, heat conduction equation, numerical solution.

Choquet integrals based on Hausdorff capacity

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Integrals of set-valued functions have been studied in connection with statistical problems and in particular with respect to convex capacities. For example, for an arbitrary set of real numbers D, we obtain some inequalities for Choquet integrals that hold w.r.t. convex capacities and non-decreasing functions.
So, if we consider the subset $D \subset \mathbb{R}^n$, $d \in (0, n]$ and we work with the d-dimensional Hausdorff capacity of D, the Choquet integral of a function $f : D \to [0, +\infty]$ with respect to H^d is then defined by

$$\int_D f(x) dH^d(x) \equiv \int_0^{+\infty} H^d(\{x \in D : f(x) > \lambda\}) d\lambda.$$

In this work we want to examine the subset of the extreme points D and in particular the case when D is the n dimensional unit cube, observing that in insurance the vertex $\mathbf{1}$ represents a full retention, while the opposite vertex $\mathbf{0}$ the full reinsurance.

Keywords: Choquet integral, Hausdorff capacity, extreme points.

Mathematical Modeling of One-Dimensional Two - Phase Oil Displacement by Steam Injection with Solvent Addition

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In this work we present the analytical solution for the problem of oil displacement by steam containing solvent as a thermal method of enhanced oil recovery. We consider one-dimensional two-phase (oil and gas) three-component (oil, solvent and water) flow in a homogeneous and isotropic porous media. Other hypotheses of the model are: incompressible system with no diffusion, no chemical reactions and no water condensation; gravity and capillary effects are neglected. Following Amagat's law, total volume is conserved and Henry's law is used to relate the solvent concentrations in gas and oil phases. The heat capacities of the components and the rock were considered constant. The dependent variables of the problem are oil saturation, solvent concentration in the oil phase and temperature. The solution, composed of shock and rarefaction waves and constant states, was obtained using the method of characteristics and analyzed for different initial and injection conditions.

Based on joint work with Adolfo Puime Pires and Fahim Forouzanfar.

Keywords: conservation laws, hyperbolic systems, thermal-solvent enhanced oil recovery.

Collocation Methods for Solving Two Dimensional Neural Field Models on Complex Triangulated Domains

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Rebecca MARTIN

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Neural field models are commonly used to describe the average activity of large populations of cortical neurons, treating the spatial domain as continuous. We present an approach for solving neural field equations that enables us to consider physiologically realistic scenarios, including complicated domains obtained from MRI data, and more general connectivity functions that incorporate, for example, cortical geometry. To illustrate our methods, we solve a popular 2D neural field model over a square domain, which we triangulate, first uniformly and then randomly. Our approach involves solving the integral form of the partial integro-differential equation directly using collocation techniques, which we compare to the commonly used method of Fast Fourier Transforms. Our goal is to apply and extend our methods to analyse more physiologically realistic neural field models, which are less restrictive in terms of the types of geometries and/or connectivity functions they can treat, when compared to Fourier based methods.

Based on joint work with J. J. Crofts, D. J. Chappell.

Keywords: Collocation Methods, Neural Field Models, Triangulated Domain.

Existence results for fuzzy integral inclusions

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To model the physical phenomena in an accurate way and to avoid from uncertainty, fuzzy integral equations/inclusions are more appropriate. In this article we use a generalized selection result of "Yannelis, Nicholas C., and N. D. Prabhakar. "Existence of maximal elements and equilibria in linear topological spaces." Journal of Mathematical Economics 12.3 (1983): 233-245." to find the existence results for certain type of fuzzy integral and integrodifferential inclusions. We extend/generalize some important results present in the literature. Some examples are given to validate the results.

Keywords: existence results, fuzzy integrals, integral inclusions, integrodifferential inclusions.

Kulkarni method for the generalized airfoil equation

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Kulkarni method is formulated and justified for solving numerically the generalized airfoil equation

$$\frac{1}{\pi}\oint_{-1}^{1}\frac{\omega(t)x(t)}{t-s}dt+\int_{-1}^{1}\omega(t)k(s,t)x(t)dt=f(s),\quad -1\leq s\leq 1,$$

where the first integral is a Cauchy principal value, $k(\cdot,\cdot)$ is a Fredholm kernel and

$$\omega(t)=\sqrt{\frac{1-t}{1+t}},\quad -1<t\leq 1,$$

using a sequence of orthogonal finite rank projections.

The convergence analysis and associated theorems are considered in this work.

Numerical examples illustrate the theoretical results and show the effectiveness of the method.

Keywords: Airfoil equation, Kulkarni method, finite rank projections.

Stochastic Boundary Integral Equations

Bas MICHIELSEN

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We present an analysis of integral equations, for time harmonic electromagnetic scattering, on stochastic boundary surfaces, $S_\alpha \subset \mathbb{R}^3$ (α coordinates on a probability space A). The solutions of such equations are singular distributions in $\mathcal{E}'(\mathbb{R}^3)$ with both stochastic values and stochastic support. We have shown that, for small deformations, an asymptotic approximation of the stochastic distribution can be given in terms of derivatives of a distribution with only stochastic values and fixed support on a "nominal surface", S_0. This distribution is the solution of a compact stochastic perturbation of a conventional integral equation on S_0. For arbitrary deformations, we get rid of the stochastic support by establishing a statistical equivalence between singular distributions, with wavefront sets in the co-normal bundles of the S_α, and a regular distribution, with fixed support $\Omega = \bigcup_{\alpha\in A} S_\alpha \subset \mathbb{R}^3$. The latter is shown to solve a volume integral equation on Ω with stochastic kernel.

Keywords: Boundary Integral Equations, Scattering, Stochastic Surfaces.

Segregated Boundary-Domain Integral Equations for Variable-Coefficient Scalar BVPs on Lipschitz Domains

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Segregated direct boundary-domain integral equations (BDIEs) based on a parametrix and associated with the Dirichlet and Neumann boundary value problems for the linear stationary diffusion partial differential equation with a variable coefficient on Lipschitz domain are considered. The PDE right hand sides belong to the Sobolev space $H^{-1}(\Omega)$ or $\widetilde{H}^{-1}(\Omega)$, when neither classical nor canonical co-normal derivatives are well defined. Although this complicates the form of the conormal derivative of the third Green identity on the boundary, it is still possible to formulate and analyse corresponding boundary-domain integral equations.

Equivalence of the BDIEs to the original BVP, BDIE solvability, solution uniqueness/non-uniqueness, as well as Fredholm property and invertibility of the BDIE operators are analysed in Sobolev (Bessel potential) spaces. It is shown that the BDIE operators for the Neumann BVP are not invertible, and appropriate finite-dimensional perturbations are constructed leading to invertibility of the perturbed operators.

Keywords: Partial differential equation, variable coefficients, parametrix.

The BMO-Dirichlet problem for elliptic systems in the upper-half space and quantitative characterizations of VMO

Dorina MITREA

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Let L be a homogeneous, second order, constant complex coefficient elliptic system L in $\mathbb{R}^n$. In this talk I will discuss the well-posedness of the Dirichlet problem in $\mathbb{R}^n_+$ for L with boundary data in $\mathrm{BMO}(\mathbb{R}^{n-1})$ in the class of functions u for which the Littlewood-Paley measure associated with u, namely $d\mu_u(x',t) := |\nabla u(x',t)|^2 \, t \, dx'dt$, is a Carleson measure in $\mathbb{R}^n_+$. The regularity result for this problem corresponding to the boundary datum being in Sarason's space $\mathrm{VMO}(\mathbb{R}^{n-1})$ will be discussed as well.

Moreover, a new characterizations of the space VMO as the closure in BMO of classes of smooth functions contained in BMO within which uniform continuity may be suitably quantified (such as the class of smooth functions satisfying a Hölder or Lipschitz condition) will be presented. This improves on Sarason's classical result describing VMO as the closure in BMO of the space of uniformly continuous functions with bounded mean oscillations.

Based on joint work with J.M. Martell, I. Mitrea, and M. Mitrea.

Keywords: BMO-Dirichlet problem, VMO-Dirichlet problem, Carleson measure.

The role of infinitesimal flatness in the solvability of elliptic boundary problems in uniformly rectifiable domains

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Marius MITREA

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The goal of this talk is to illustrate the phenomenon that uniform rectifiability together with infinitesimal flatness (understood as the demand that the outward unit normal is close to having vanishing mean oscillations) typically implies solvability results for elliptic boundary value problems formulated in such a geometric setting.

Keywords: elliptic boundary problems, uniform rectifiable sets, vanishing mean oscillations.

Composites with invisible inclusions: eigenvalues of $\mathbb{R}$-linear problem

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Vladimir MITYUSHEV

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A new eigenvalue $\mathbb{R}$-linear problem arisen in the theory of metamaterials and neutral inclusions is reduced to integral equations. The problem is constructively investigated for circular non-overlapping inclusions. An asymptotic formula for eigenvalues is deduced when the radii of inclusions tend to zero. The nodal domains conjecture related to univalent eigenfunctions is posed. Demonstration of the conjecture allows to justify that a set of inclusions can be made neutral by surrounding it with an appropriate coating.

Keywords: $\mathbb{R}$-linear problem, metamaterials, univalent eigenfunction.

Nonlinear boundary-domain integral equations of boundary value problems for scalar quasilinear elliptic PDEs

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David NATROSHVILI

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Non-localized and localized nonlinear boundary-domain integral equation (NBDIE) formulations of the Dirichlet, Neumann and Robin boundary value problems are considered for some quasilinear partial differential equations of elliptic type. Using the properties of the non-localized and localized layer and volume potentials it is shown that the NBDIE systems are equivalent to the original classical and/or weak setting of the boundary value problems. For some special type of nonlinearities the corresponding NBDIE systems are analyzed in detail and an iterative method convergence is proved employing the Banach fixed-point theorem.

Based on joint work with Sergey Mikhailov.

Keywords: quasilinear partial differential equations, nonlinear boundary-domain integral equations.

The boundary value contact problem of electro - elasticity for piecewise-homogeneous piezo - elastic plate with inclusion and cut

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The problem of determining the mechanical and electric fields in a piecewise-homogeneous piezo-electric medium weakened with a semi-infinite cut and supported by a finite elastic inclusion is considered. Using the methods of theory of analytic functions the problem reduces to a system of singular integro-differential equations with fixed singularity. The exact solution of the posed problem is obtained by integral Fourier transformation.

Keywords: Contact problems, Piezo-elastic plate, Cut.

Numerical Solution of Free Surface Flow Modeled with Shallow Water Equations Using Finite Volume Method in Conjuction with Godunov Type Riemann Solvers

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Nuray ÖKTEM

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In this study, a computer code is developed for the solution of flow around an obstruction like bridge piers. The problem is modeled using the two-dimensional depth integrated nonlinear shallow water equations (NSWE). The numerical approach is based on a finite volume method (FVM) involving HLLC Riemann solver at each cell face. Additionally, a total variation diminishing (TVD) version of weighted averaged flux (WAF) method is employed for interface

flux computations to ensure the stabilization and accuracy of the solutions. For time integration a two-stage Runge-Kutta method is utilized. Thus, this method is capable of handling discontinuities due to shock waves and therefore able to capture wave propagation at interfaces with high resolution. The model is performed for various test cases and the numerical results are presented.

Based on joint work with İsmail Aydın.

Keywords: Shallow flow, Riemann solvers, finite volume.

Monoenergetic neutron space-kinetic equation in full cylinder symmetry with axial heterogeneity: A novel analytical solution

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In the present work we discuss mono-energetic neutron space-kinetics with one group of delayed neutron precursors in cylindrical geometry, for two sectionally homogeneous cylinder cells with distinct nuclear parameters. This equation is self-adjoint for a mono-energetic model and a homogeneous cylinder. By the use of variable separation the associated spectra are analysed in order to determine a solution. In a subsequent step continuity in the scalar flux as well as in the current density at the sub-domain interface establishes the global solution. The interface approximation due to control rod insertion is based on comparing the time scale typical for the boundary change that is very much larger than the neutron life time so that changes in the neutron flux are slower and a quasi-static problem for the boundary is assumed. We present some numerical results for the scalar neutron flux and neutron precursors with time dependence and spatial projections.

Based on joint work with Bardo E.J. Bodmann, Fernando Carvalho, Marco T. Vilhena.

Keywords: Neutronics, space kinetics, analytical solution.

A Semi-Analytical Solution for a Build-up Test for a Horizontal Well in an Anisotropic Gas Reservoir

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Adolfo PIRES

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This work presents an approximate semi-analytical solution for a horizontal well in an anisotropic gas reservoir by means of the Green's Function technique. First, by changing the dependent variable, the nonlinear hydraulic diffusivity gas equation becomes a quasi-linear partial differential equation. Next, the formulation is rewritten as a linear differential equation with a nonlinear source, which is then recast as a nonlinear Volterra-Fredholm integral equation of the second kind. An approximate solution is proposed which considers just one Picard iteration and evaluation of the integrals by multi-dimensional numerical integration. This approximate solution is applied to horizontal gas wells producing under a variable-rate scheme often seen in gas field operations, and shows close agreement to the results of a commercial finite-difference reservoir simulator. The approximate solution is then applied to study the non-linear behavior shown by buildup tests on horizontal gas wells in anisotropic reservoirs.

Based on joint work with Bruno J. Vicente and Alvaro M. M. Peres.

Keywords: horizontal well, gas reservoir, Green's functions.

Kinect Depth Recovery based on Local Filters and Plane Primitives

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With the advent of RGB-D cameras, such as Kinect, it is now possible to benefit from both color and depth of the field of view in real-time towards a better understanding of the scene. While such devices are providing appreciative depth information; captured depth map suffers from noticeable noise. Recent methods reach satisfactory results in depth recovery by focusing on color and depth images individually or cooperatively. In this paper, we propose a geometric approach to structurally model the scene by extracting a series of planes from the point cloud. The problem is formulated as an energy minimization function based on initial depth values calculated by modelling the scene using planes, and applying local filters on color image and depth map. The presented method is implemented and tested on simulation data and experimental results show its accurate and precise performance.

Based on joint work with Mahdi Abolfazli Esfahani.

Keywords: RGB-D camera, Depth Recovery, RANSAC, Point Cloud.

A metaheuristic approach for an optimized design of a silicon carbide operational amplifier

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Silicon Carbide (SiC) has shown to be a superior material compared to Silicon for high temperature integrated electronics applications, due to its excellent physical properties. In SiC

electronics, device parameters' variations over the temperature range along with the fabrication process variations has made the optimized design of analog integrated circuits extremely challenging. A fundamental analog integrated circuit is an operational amplifier, for which voltage gain is considered as a significant performance metric. In this paper, a metaheuristic is presented to optimize voltage gain in the design of a bipolar SiC operational amplifier. The problem is analyzed in the existence of different design and fabrication process constraints and variables. The results of the algorithm are also compared with the voltage gain reported in the experimental case study. In future works, the method can also be extended to fit for other analog circuits in SiC and similar emerging semiconductor technologies.

Based on joint work with Saleh Kargarrazi, KTH Royal Institute of Technology, Sweden.

Keywords: Optimization, Silicon Carbide, Operational Amplifier.

Application of Stochastic Dynamic Programming in Demand Dispatch-based optimal operation of a Micro-gri

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in the field of mathematical optimization, stochastic dynamic programming (SDP) is a good framework to model uncertain optimization problems. In power system, Demand dispatch (DD) is a new emerging operation concept compatible with uncertain renewable generations. In the conventional operation concept called load following, generation follows the load variations through economic dispatch program which is actually supply dispatch. But, to operate a system with a high penetration of stochastic generation units, it would be better to apply demand dispatch with the paradigm of generation following. In this paper for the first time we seek to present DD problem through clear formulations based on SDP of dispatchable loads. If the operation horizon, here a day, is divided by N time periods, the operator tries to find the chain of optimal control law so as to minimize the expected total cost. The operator selects the optimal control strategy for every time period.

Based on joint work with Fateme Daburi Farimani.

Keywords: uncertain optimization problems, stochastic dynamic programming, demand dispatch.

Asymptotic expansions of iterated Galerkin solution of second kind Fredholm integral equations with Green's type kernels

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Consider the second Fredholm integral equation of the second kind

$$u(s) - \int_0^1 k(s,t)u(t)dt = f(s), \quad s \in [0,1],$$

where the kernel is of the Green's function type. We consider the iterated Galerkin method for the approximate solution of the above equation. We consider the approximating space of piecewise polynomials of degree $\leq r$ with respect to a uniform partition of $[0,1]$ having n subintervals. Let $h = \frac{1}{n}$. We obtain asymptotic expansion for the iterated Galerkin solution u_n at the partition points $\frac{i}{n}$, $i = 0, \ldots, n$. We then use Richardson extrapolation to obtain approximate solutions with higher orders of convergence. Numerical results confirm the theoretical orders of convergence. These results can be extended to nonlinear equations.

Keywords: Green's function type kernels, Iterated Galerkin method, Asymptotic expansion.

New operational relations and applications of a fractional Fourier transform

Manuela RODRIGUES

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In this talk, we deal with the fractional Fourier transform in the form introduced a little while ago by Yuri Luchko. This transform is closely connected with the FC operators and has been already employed for solving of both the fractional diffusion equation and the fractional Schrödinger equation. In this work, we continue investigation of the fractional Fourier transform and in particular prove some new operational relations for a linear combination of the left- and right-hand sided fractional derivatives. As an application of the obtained results, we provide a schema for solving the fractional differential equations with both left- and right-hand sided fractional derivatives without and with delays and give some examples of realization of our method for several fractional differential equations.

Based on joint work with Yuri Luchko.

Keywords: Fractional Fourier transform, left- and right-hand sided fractional derivatives, fractional differential equations.

Severe precipitation evaluation in Brazil: Data mining approach

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Heloisa M. RUIVO

National Institute for Space Research (INPE), Brazil

Data mining approach is applied to evaluate extreme rainfall events in the Brazil. Statistical analysis is combined with an artificial intelligence technique to identify the most relevant meteorological variables for a local severe precipitation in the Rio de Janeiro state (Brazil): Rio de Janeiro, and Nova Friburgo cities. The p-value statistical technique is employed to select a much smaller subset of climatic variables, preserving the information associated with extreme meteorological events. A decision tree algorithm is used as a model to identify the precipitation severity. The method is tested with the events at Apr/2009 (Rio de Janeiro city) and at Jan/2011 (Nova Friburgo city). In both cases, our results show a good local analysis for extreme precipition episodes.

Based on joint work with Haroldo F. Campos Velho, and Saulo R. Freitas.

Keywords: Severe precipitation, statistical p-value analysis, decision tree algorithm.

Theory of plasmonic metasurfaces

Matias RUIZ

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We consider the scattering of electromagnetic waves by a layer of periodic plasmonic nanoparticles mounted on a perfectly conducting sheet. We design the layer in order to control and transform waves. Since the thickness of the layer is negligible compared to the wavelength, it can be approximated by an impedance boundary condition. Our main result is to prove that at some resonant frequencies, the thin layer has anomalous reflection properties and can be viewed as a metasurface. The resonant frequencies of the array of nanoparticles differ significantly from those of a single nanoparticle as they are associated with eigenvalues of a periodic Neumann-Poincaré type operator which depends on the particle size and relative position.

Our main findings open a door for a mathematical and numerical framework for realizing full control of waves using metasurfaces and for explaining and optimizing the observed extraordinary or meta properties of such structures.

Based on joint work with Habib Ammari, Wei Wu, Sanghyeon Yu and Hai Zhang.

Keywords: plasmonic resonance, metasurfaces, periodic Neumann-Poincaré operator.

A condition for the existence of trapped modes

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It is known that a waveguide with cylindrical outlets into infinity may have trapped modes. In [Evans D.V., Levitin M. and Vassiliev D., *J. Fluid Mech.* **261** (1994), 21–31], it was proved that a symmetric (finite) obstacle placed in the symmetry axis of a two-dimensional waveguide supports a trapped mode, in this case an embedded eigenvalue, in the continuous spectrum. Since then a several examples on the existence of trapped waves have been demonstrated in various waveguides.

In this short presentation we will investigate the conditions under which the waveguide has a trapped mode. We consider the problem on a channel which consists of a finite part and two cylindrical outlets into infinity. The finite (bounded) part may contain some submerged or/and surface piercing bodies. Since the ordinary scattering matrix can by no means contribute any information on trapped modes, we introduce the fictitious scattering operator and present a criterion for the existence of trapped modes. The criterion states that the number of trapped modes is the difference of the multiplicities of the eigenvalue 1 of the fictitious scattering operator and the eigenvalue $-i$ of the scattering matrix.

Based on joint work with Sergei A. Nazarov.

Keywords: trapped modes, criterion, fictitious scattering operator.

Performance Assessment of a New FFT Based High Impedance Fault Detection Scheme

Javad SADEH

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High Impedance Faults (HIFs) are commonly known for their unpredictable, non-linear and unstable nature, due to the presence of electric arcs. Considering the low current amplitude of these faults, reliable detection and discrimination from typical power system events have proven to be troublesome. Conventional overcurrent relaying techniques are insensitive to the similar behavior of these faults and single phase loads. Extraction of harmonic patterns from FFT spectrum as HIF signature, along with joint execution of abnormal condition detection, have led to the design of a new FFT based scheme. In this paper, an in depth performance evaluation has been executed to verify its functionality. Simplicity, low computational burden and high accuracy in successful fault and switching event detection are some of its virtues.

Simulation of an IEEE standard distribution system and performance evaluations in regards to conventional methods have provided proof to this claim.

Based on joint work with A. Soheili.

Keywords: Fourier transform, harmonic analysis, high impedance fault.

Shifting the boundary conditions to the middle surface in the numerical solution of Neumann boundary value problems using integral equations.

Alexey SETUKHA

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The 3-D Neumann boundary value problem for the Laplace equation out of the body of small thickness is considered. An approach is proposed to solve the problem in which the boundary conditions are transferred to the middle surface of the body. As a result, a new boundary value problem on the screen (the middle of the body surface) is solved. The initial shape of the body is taken into account using the corresponding boundary conditions. The resulting boundary value problem on the screen is reduced to a system of boundary integral equations with singular and hypersingular integrals. These integral equations are solved numerically using the methods of piecewise constant approximations and collocation.

On the basis of the developed approach a new version of the panel method for solving the problem of flow past wing is built using the wing replacement with the middle surface. The numerical tests showed that the reduction of the wing thickness does not decrease the accuracy of the numerical results.

The proposed method is applied to the problem of flow around a gliding parachute and the calculation of its shape.

Keywords: Integral equations, hypersingular integrals, panel methods.

The two-dimensional integro-differential equations and their applications in the theory of viscoelasticity

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Nugzar SHAVLAKADZE

A. Razmadze Mathematical Institute, Iv.Javakhishvili Tbilisi State University, Georgia

The effective solutions for integro-differential equations related to problems of interaction of an elastic thin finite inclusion with a plate, when the inclusion and plate materials possess the creep property are constructed. If the geometric parameter of the inclusion is measured along its length according to the parabolic and linear law we have managed to investigate the obtained boundary value problems of the theory of analytic functions and to get exact solutions and establish behavior of unknown contact stresses at the ends of an elastic inclusion.

Keywords: Integro-differential equations, Viscoelasticity, Finite inclusion.

Droplet Deposition and Coalescence in Curved Pipes

Ovadia SHOHAM

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A novel and systematic experimental investigation was conducted on droplet deposition and coalescence in curved pipes. Various curved pipe bends, to be utilized as flow conditioning devices upstream of wet gas separators, were tested. Data were collected for about 200 experimental runs, varying v_{ratio} up to 4.5, for two liquid loadings of 700 and 1400 $m^3/MMsm^3$. The results show that the 180^o pipe bend and the long elbow bend performances are similar, which are the best among the curved pipes tested. The long elbow bend is recommended for field applications for its performance, availability, and ease of installation. A model for the prediction of droplet deposition in a long elbow bend was developed, based on the physical phenomena, consisting of a force balance and a conservation of angular momentum on a droplet. The model enables tracking the droplet movement in the elbow and identifying the depositing droplets. A comparison between the model predictions and experimental data shows a good agreement with average error about 20.

Based on joint work with H. Nguyen, R. Mohan and G. Kouba.

Keywords: droplet deposition, curved pipes.

An effective tool for solving integral equations via fixed point theorems

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Deepak SINGH

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In recent times, fixed point theory is utilized in many other areas of mathematics, such as ordinary and partial differential equations, Integral equations and in optimization techniques as an important tool for solving problems, arising in these settings. In my talk, I will stress on the applications of fixed point theorems to solve integral equations and boundary value problems by acknowledging some of the recent papers [Filomat. 28 (10) (2014) 2047–2057, Fixed Point Theory and Applications, Volume 2010, Article ID 621469, 17 pages, Mediterr. J. Math., 2015, DOI 10.1007/s00009-014-0506-y] in various abstract spaces. Moreover, some fixed point results are established which are focused on mainly the applications for existence of the solution of various Integral equations and Boundary value problems. This demonstrates the superiority of fixed point theorems and its applications.

Keywords: Fixed point, Integral equation, Boundary value problem.

A Navier type wall-law for an incompressible flow in a rough pipe

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Maja STARČEVIĆ

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We observe an incompressible viscous fluid flow through a pipe which lateral boundary is consisted of periodically distributed asperities. The flow is governed by incompressible linearized Navier-Stokes equations. The uniform pressure drop is assumed, i.e. we pose constant pressures on the inflow and outflow boundary. The normal velocity at the nonlateral boundary is assumed as well.

The aim is to approximate the boundary condition on a smooth artificial boundary near the lateral one. To do this, we transform the pipe into a smooth one and afterwards we perform the asymptotics steps on the modified pipe. Following formal expansion we obtain an approximation of the flow that is confirmed in an appropriate norm. Using it, a Navier type effective boundary condition is obtained.

Based on joint work with Eduard Marušić-Paloka.

Keywords: pipe flow, rough boundary, effective boundary condition.

Fast parallel algorithms for solving integral equations in boundary value problems of diffraction

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Stanislav STAVTSEV

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Problems of diffraction on complex-shaped objects are often solved by integral equations. The numerical solution of integral equations is characterized by the presence of dense matrices of size $10^5 \dots 10^7$. In this study, a well-scalable (up to several hundreds of processors) parallel algorithm is used to construct $\mathcal{H}^2$-approximations for the dense matrices.

If the integral equation operator is hypersingular, the numerical solution of the integral equation with a large wave number requires a large number of iterations. In this study, a parallel direct solver for systems with a $\mathcal{H}^2$-matrix was developed. The algorithm is based on the representation of the $\mathcal{H}^2$-matrix as a block-tridiagonal matrix and the use of the MUMPS package

(MUltifrontal Massively Parallel Solver, http://gral.ens-lyon.fr/MUMPS) to parallelize the algorithm of sparse matrix inversion.

The work was supported by the Russian Science Foundation, grant 14-11-00806.

Keywords: scattering problems, fast algorithms, direct solvers.

Influence of dry deposition velocity to the ground in modeling of dispersion of the air pollutants

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Tiziano TIRABASSI

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We describe the pollutant dispersion in the atmosphere considering the dry deposition velocity to the ground. Since some pollutants tend to deposit on the earth surface, it is appropriate modify the atmospheric diffusion models so that they consider the loss of pollutants by dry deposition. The solution of the three-dimensional stationary advection-diffusion equation, which describes dry deposition as a boundary condition of nonzero flow at the ground level, is obtainable through the GIADMT approach (Generalized Integral Advection-Diffusion Multilayer Technique). This method combines the GITT approach that is applied in the cross-wind direction and the transformed problem is solved by the ADMM method, thus obtaining an analytical integral solution based on the Laplace transform technique. Simulations of the dispersion of pollutants in different turbulent scenarios are performed, and the values predicted by the solution are compared with experimental data, then the effect of dry deposition on pollution concentrations is analyzed.

Based on joint work with C. P. Costa and K. Rui.

Keywords: Advection-Diffusion Equation, Air Pollution Dispersion, Dry Deposition.

Spectral Tau method for systems of nonlinear integro-differential equations

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In this paper an extension of the Lanczos' tau method to systems of nonlinear integro-differential equations is proposed. This extension includes (i) linearization coefficients of orthogonal polynomials products issued from nonlinear terms and (ii) recursive relations to implement matrix inversion whenever a polynomial change of basis is required. Both these improvements ensure numerical stability and accuracy in the approximate solution. Furthermore, these techniques are also explored in the context of piecewise approximate solution of integro-differential equations to tackle problems over large intervals. Exposed in detail, this

novel approach is able to significantly outperform numerical approximations with other methods as well as different tau implementations. Numerical results on a set of problems illustrate the impact of the introduced mathematical techniques.

Based on joint work with José M. A. Matos (Instituto Superior Engenharia do Porto and Centro de Matemática da Universidade do Porto, Portugal) and Paulo B. Vasconcelos (Faculdade Economia da Universidade do Porto and Centro de Matemática da Universidade do Porto, Portugal).

Keywords: Spectral methods, tau and segmented tau methods, integro-differential equations.

On The Dynamics And Periodicity Of Solutions Of Higher Order Nonlinear Difference System

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In this study, we examine the solutions of a higher order differnce system. We, first, obtain equilibrium points of the system. Then, we find the periodicity of solutions and investigate the Dynamics of this system related to equilibrium points.

Based on joint work with G. Salli.

Keywords: difference equation, difference system, equilibrium points.

On the inversion of a generalized Radon transform

Zekeriya USTAOGLU

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We consider a generalized Radon transform (GRT) that integrates a function $f(x_1, x_2)$ on $\mathbb{R}^2$ over a family of curves $x_2 = u + s\varphi(x_1 - c)$ with respect to the variable x_1, for a real valued continuous function φ on $\mathbb{R}$, $u, s \in \mathbb{R}$ and a fixed $c \in \mathbb{R}$. This transform can be regarded as a generalization of the slant-stack transform in seismology.

We investigate the inversion of the GRT via the inversion of the regular Radon transform. Depending on some conditions on f and φ, we obtain some back-projection type inversion formulas and also describe a method for numerical reconstruction of f from its GRT. Numerical results are presented to demonstrate the feasibility of the proposed method.

Keywords: Generalized Radon transform, Inversion formula, Numerical reconstruction.

An efficient method to solve a fractional differential equation

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This paper presents a new idea to solve fractional differential equations and fractional optimal control problems. The fractional derivative is defined in the Grunwald–Letnikov sense. The method is based on the linear programming problem. In this paper, by using first the concept of fractional derivatives, we will suggest a method where an equation with a fractional derivative is changed to a linear programming problem, and by solving it the fractional derivative will be obtained. Actually this suggested method is based on the minimization of total error. Some numerical examples are provided to confirm the accuracy of the proposed method.

Based on joint work with Ali Rakhshan and Sohrab Effati.

Keywords: Nonlinear mixed Volterra-Fredholm integral equations, Periodic quasi-wavelet, Complex plane, Fixed point theorem.

Pseudo-differential equations and related continual and discrete boundary value problems

Vladimir VASILYEV

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Lipetsk, Russia

We discuss different formulations of boundary value problems for elliptic pseudo differential equations and relationships between them. Considering these problems in corresponding Sobolev-Slobodetskii spaces we extract some common properties for discrete and continual cases.

Key concepts of this approach are the local principle and the wave factorization. We consider these principles both for a continual situation and a discrete one. In this way a Fredholm properties of boundary value problems for elliptic pseudo-differential equations on manifolds with non-smooth boundaries can be described.

Keywords: pseudo-differential equation, wave factorization, boundary value problem.

A poroelastic model of spinal cord cavities

Jenny VENTON

School of Computing, Engineering and Mathematics, University of Brighton, UK

Syringomyelia is a rare disease characterised by macro fluid filled cavities in the spinal cord. The process through which these cavities form is not fully understood, although it is thought to be influenced by pressure changes in the fluid surrounding the cord.

Using a poroelastic material model for the spinal cord tissue, a finite element model of the spinal cord is being built. This will investigate the effect that pressure changes in the surrounding fluid have on the spinal cord tissue. For example, how internal cord fluid pressure is affected and what stresses this could exert within the spinal cord.

The model is being updated with elasticity, permeability and porosity parameters derived from experimental work and existing studies. The updated model will demonstrate whether the stresses that occur within tissue are enough to damage the cord and contribute to cavity formation.

Based on joint work with Paul Harris and Gary Phillips.

Keywords: finite element method, poroelastic, syringomyelia.

Fundamental solutions for 3-dimensional fractional Laplace and Dirac operators

Nelson VIEIRA

CIDMA - Center for Research and Development in Mathematics and Applications, Department of Mathematics, University of Aveiro, Portugal

The aim of this talk is to present, by application of two methods, a closed formula for the family of eigenfunctions and fundamental solutions of the three-parameter fractional Laplace operator using Riemann-Liouville fractional derivatives, as well as, a family of fundamental solutions of the associated fractional Dirac operator.

The first approach corresponds to an application of operational techniques via the Laplace transform associated to the extension of the Laplace transform to generalized functions. In the second approach we consider the classical method of separation of variables. In both approaches the obtained solutions are expressed in terms of Mittag-Leffler functions.

Since there is a duality relation between Caputo and Riemann-Liouville fractional derivatives, we present the analogous of the presented results for the case where Caputo fractional derivatives are used. Moreover, we present some graphical representations of the solutions obtained via separation of variables for some values of the fractional parameter.

Based on joint work with M. Ferreira (CIDMA, University of Aveiro, Portugal).

Keywords: Eigenfunctions and fundamental solution, Fractional Laplace and Dirac operators, Riemann-Liouville derivatives and integrals of fractional order.

Existence of Frequency Modes Coupling Seismic Waves and Vibrating Tall Buildings

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Darko VOLKOV

Worcester Polytechnic Institute

We prove an existence result for frequency modes coupling seismic waves and vibrating tall buildings. The derivation from physical principles of a set of equations modeling this phenomenon was done in previous studies. In this model all vibrations are assumed to be anti-plane and time harmonic so the two dimensional Helmholtz equation can be used. A coupling frequency mode is obtained once we can determine a wavenumber such that the solution of the corresponding Helmholtz equation in the lower half plane with relevant Neumann and Dirichlet at the interface satisfies a specific integral equation at the base of an idealized tall building. Although numerical simulations suggest that such wavenumbers should exist, as far as we know, to date, there is no theoretical proof of existence. We also present numerical results on computing these coupling frequencies. In the case of a large number of buildings, our numerical method using periodic domains and periodic Green's functions yields much faster computations. This is the main reason why we are now able to study systems of buildings of variable height, mass, and rigidity. Interestingly, in the case of non-identical buildings, our simulations indicate that the response to this coupling phenomenon may differ drastically from one building to another.

Based on joint work with S. Zheltukhin.

Keywords: Integral equations on an open interval, low and high frequency asymptotics

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Counter-gradient term applied to the turbulence parameterization in the BRAMS

Maria Eugênia S. WELTER

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The numerical weather prediction is a routine in operational meteorological centers, where sophisticated computer models are runned. The atmospheric dynamics is simulated by solving the Navier-Stokes equation, considering several physical phenomena. One parameterization is to represent the turbulence. A counter-gradient flow can be described for higher order closure turbulence approaches. Here, a first order parameterization for the turbulent flow is coupled with an explicit counter-gradient term. Both latter schemes are based on the Taylor's statistical theory of turbulence. These new schemes are applied to the BRAMS, a meso-scale meteorological model. The simulation is compared with experimental data measured in the Brazilian Amazon region.

Based on joint work with Haroldo F. Campos Velho, and Saulo R. Freitas.

Keywords: turbulence parameterization, counter-gradient term, BRAMS model.

A scalable two-stage Fast Multipole Method preconditioner for coupled flow and transport simulation

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Rami M. YOUNIS

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The continuity of mass equations that govern the flow of multiple fluid phases in porous media are a system of coupled nonlinear PDEs. The two independent state variables are pressure and saturation. While pressure evolves with a parabolic character, saturations evolve as traveling waves with a finite domain of dependence. Fully implicit finite volume methods are often used to numerically approximate these equations. This results in a coupled nonlinear algebraic system of equations that must be solved at each timestep. Newton-like methods are employed, requiring the solution of linear algebraic systems at each iteration. We propose a two-stage preconditioning strategy that treats the elliptic component in the first stage using a Fast Multipole Method that approximately solves a boundary integral equation. The integral equation arises by linearizing the PDE in functional space. Compared to traditional ILU-GMRES strategies, we demonstrate superior convergence.

Based on joint work with Saman Aryana.

Keywords: two-stage preconditioner, porous media, fast multipole method.

Time and space splitting methods for systems of partial functional differential equations modeling tumor growth

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Barbara ZUBIK-KOWAL

Department of Mathematics, Boise State University

This talk is devoted to splitting methods and a fast algorithm constructed to compute approximate solutions to systems of partial functional differential equations used in modeling the growth of human tumor cells and their response to therapy in a parallel computing environment. The construction of the algorithm is based on a decomposition of the time domain and on a reassignment of the spatial grid-points that depends on the given number of available

processors. Theoretical results validating the robustness of the algorithm will be presented together with a series of numerical experiments across different numbers of independently working processors.

Keywords: partial functional differential equations, growth of human tumor cells, numerical solutions.

SECTION 3

Minisymposia

Asymptotic Analysis: Homogenization and Thin Structures

Organizer: M. Eugenia Pérez (Universidad de Cantabria, Santander, Spain)

Homogenization of materials with sign changing coefficients

Renata BUNOIU

IECL, UMR 7502 CNRS, University of Lorraine, Metz, France,

We investigate a periodic homogenization problem involving two isotropic materials with conductivities of different signs: a classical material and a metamaterial - or negative material. Combining the $\mathbf{T}-$coercivity approach and the unfolding method for homogenization, we prove well-posedness results for the initial and the homogenized problems and we obtain a convergence result. These results are obtained under the condition that the contrast between the two conductivities is large enough in modulus. The homogenized matrix, is generally anisotropic and indefinite, but it is shown to be isotropic and (positive or negative) definite for particular geometries having symmetries.

Based on joint work with Karim Ramdani.

Keywords: Metamaterials, $\mathbf{T}-$coercivity, periodic unfolding.

Uniform resolvent convergence for a planar strip with fast oscillating boundary

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Giuseppe CARDONE

Department of Engineering, University of Sannio, Benevento, Italy

We consider an elliptic operator in a planar infinite strip perturbed by substituting one side of the boundary by a fast oscillating curve. We assume that both the period and the amplitude of the oscillations are small and impose the Dirichlet condition on the upper boundary and Dirichlet, Neumann or Robin boundary condition on the oscillating boundary. In all cases we describe the homogenized operator, establish the uniform resolvent convergence of the perturbed resolvent to the homogenized one, and prove the estimates for the rate of convergence. These results are obtained as the order of the amplitude of the oscillations is less, equal or greater than that of the period. It is shown that under the homogenization the type of the boundary condition can change.

Based on joint work with D.Borisov, L.Faella, C.Perugia.

Keywords: Homogenization, Uniform resolvent convergence, Oscillating boundary.

Nematic Liquid Crystals

Gregory A. CHECHKIN

Department of Differential Equations, Faculty of Mechanics and Mathematics, Lomonosov Moscow State University, Russia

We consider the full 3D Ericksen–Leslie system and prove the existence of the unique solution in periodic as well as in a bounded domain.

Also we study the homogenization problem for a system of equations describing the dynamics of a mixture of liquid crystals with random structure. We consider a simplified form of the Ericksen–Leslie equations for an incompressible medium with inhomogeneous density with random structure. Under the assumption that randomness is statistically homogeneous and ergodic, we construct the limit problem and prove almost sure convergence of solutions of the original problem to the solution of the limit (homogenized) problem.

Based on joint work with Tatiana P. Chechkina, Tudor S. Ratiu, Maxim S. Romanov, Vyacheslav N. Samokhin.

Keywords: nematic liquid crystals, homogenization, existence and uniqueness of solutions.

Homogenization for the p-Laplace operator in perforated media along a manifold.

Delfina GÓMEZ

Departamento de Matemáticas, Estadística y Computación, Universidad de Cantabria, Spain

We address a homogenization problem for a variational inequality posed in a perforated domain of $\mathbb{R}^n$, with $n \geq 3$, issue from unilateral problems of the p-Laplacian when $p \in [2, n)$. ε is a small parameter which measures the periodicity of the structure while $a_\varepsilon \ll \varepsilon$ measures the size of the perforations, which are placed along a $n-1$ manifold inside Ω. Nonlinear Robin type conditions (associated with the p-Laplacian) are imposed on the boundary of the cavities involving a large parameter. We derive the homogenized model depending on the relations between the parameters of the problem.

Based on joint work with M. Lobo, E. Pérez, A.V. Podolskii, T.A. Shaposhnikova.

Keywords: nonlinear homogenization, perforated media, variational inequalities.

Stochastic homogenization of non-local Hamilton-Jacobi equations

Ahmed HAJEJ

Ceremade, University of Paris-Dauphine, France

We study the homogenization of non-local Hamilton-Jacobi equations in stationary ergodic settings. These equations can be seen as a level-set approach of front propagation problems that move in the normal direction with non-local velocities. We first identify the effective Hamiltonian and an approximate corrector by studying the associated metric problem. Then, we prove the main homogenization result by means of a non-local version of the perturbed test function method.

Keywords: Stochastic homogenization, non-local Hamilton-Jacobi equations, viscosity solution.

Outer-plane properties of thin heterogeneous periodic layers made of beams

Julia ORLIK

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The elasticity problem for two domains separated by a thin heterogeneous layer with a periodic in-plane structure (the period of the structure $\varepsilon \ll 1$) is studied. The layer of the thickness δ consists of the thin beams of radius r. The limit behavior of this problem is studied as the period ε, the thickness δ and the radius r of the beams tend to zero. The decomposition of the displacement field in the beams is used, which allows to obtain component-wise a priori estimates.

An asymptotic analysis with respect to $\varepsilon, r, \delta \to 0$ is provided and the limit elasticity problem together with the Robin transmission condition across the interface is obtained. The periodic unfolding method is used to study the limit behavior. A numerical example is presented to illustrate the results.

Based on joint work with G. Griso and A. Migunava.

Keywords: homogenization, dimension reduction, thin structures.

Asymptotic solutions of Maxwell equations in layered periodic structure

Maria PEREL

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Monochromatic electromagnetic waves in layered periodic structure are studied asymptotically. The frequency is assumed to be close to a stationary point of the dispersive surface. Two-scale asymptotic expansions method yields the field in the principal order as a sum of two Floquet-Bloch solutions of different polarizations multiplied by a slowly changing envelope functions. A system of differential equations for envelope functions is derived. The system is split in two independent equations only if the field does not depend on one of spatial coordinates.

Based on joint work with M. Sidorenko.

Keywords: periodic structure, two-scale expansions, formal asymptotics.

Boundary homogenization in perforated domains for adsorption problems

Maria Eugenia PÉREZ

Departamento de Matemática Aplicada y Ciencias de la Computación, Universidad de Cantabria, Spain

We consider a model for the spreading of a substance through an incompressible fluid in a perforated domain Ω_ε, $\Omega_\varepsilon \subset \Omega \subset \mathbb{R}^3$. The fluid flows in a domain containing a periodical set of perforations $(\Omega \setminus \Omega_\varepsilon)$ placed along an inner surface $\Sigma \subset \Omega$. The size of the perforations is much smaller than the size of the characteristic period ε. An adsorption phenomena can occur on the boundaries of the perforations, where we assume a strongly nonlinear adsorption law with a large adsorption parameter. We derive the homogenized model depending on the relations between the parameters of the problem.

Based on joint works with D. Gómez, M. Lobo and T. A. Shaposhnikova.

Keywords: boundary homogenization, porous media, critical relations.

Singular problems with nonlinear Robin conditions in perforated domains

Federica RAIMONDI

Laboratoire de Mathématiques Raphaël Salem, University of Rouen, France
Department of Mathematics, University of Salerno, Italy

The talk is devoted to show the existence and uniqueness of the solution of a quasilinear elliptic problem with a singular nonlinearity, posed in a perforated domain.
More precisely, the quasilinear equation presents a term which is singular in the u-variable (u being the solution), which is a product of a continuous singular function and another one whose summability depends on the growth of the first one near its singularity.
The difficulty arises in dealing simultaneously with the matrix field depending on the solution itself, the singular datum and the nonlinear Robin condition.
Firstly, we prove some a priori estimates, crucial for proving the existence and uniqueness result. Then, as usual in the literature, we find a solution by approximating our problem with a sequence of nonsingular problems with a bounded nonlinearity.
For uniqueness of the solution, some additional hypotheses on the singular function and the matrix field are required.

Based on joint work with P. Donato and S. Monsurrò.

Keywords: quasilinear, singular, perforated.

Averaging of the problems for the p – Laplace operator in perforated domains with nonlinear boundary conditions involving a large parameters

Tatiana SHAPOSHNIKOVA

Department of Differential Equations, Moscow State University, Russia

We consider the homogenization of the boundary-value problems for the p – Laplace operator in a perforated domain. On the boundary of perforations we have nonlinear boundary conditions or restrictions for the solutions and their flux. Supposing that $1 < p < n$ we obtain the homogenized problems for all the relations between different parameters of the problem.

Based on joint work with D. Gómez, E. Pérez, A.V. Podolskii.

Keywords: nonlinear homogenization, p – Laplace, perforated domains.

Asymptotic Solutions of Short Wave-Forms of the Modified Scalar Wave Equation Under Discretization, In Multidimensions

John STEINHOFF

Wave CPC, Inc.

A simple term that obeys an eigenvalue condition, when added to the scalar wave equation, can result in stable nonlinear solitary waves. These modified waves remain thin and preserve the shape asymptotically, even under perturbations of the original partial differential equation (pde) due to discretization. These waves pass through each other with no phase-shift, while preserving their individual properties. They obey the original wave equation and can reflect from surfaces that are immersed in uniform Cartesian grids. This idea has been proven useful in capturing short pulses or vortex filaments without discretization errors.

Based on joint work with Subhashini Chitta.

Keywords: Wave equation, solitary waves, short waves.

Multiharmonic analysis for nonlinear acoustics with small excitation amplitude

Anastasia THÖNS-ZUEVA

DFG research center MATHEON, TU Berlin

Acoustic experiments show that the noise absorption by perforated walls differs if small or large acoustic amplitudes are used. In addition, an interaction between different frequencies has been observed. To describe these effects we consider the compressible Navier-Stokes equations in time domain with the nonlinear advection term, which couples velocity and pressure. Here, we consider small amplitudes of the excitation and viscosities like $O(\varepsilon^2)$ with some small parameter ε. Nonlinear acoustics can be described in frequency domain by the multiharmonic analysis applied to the compressible Navier-Stokes equations, which takes into account the frequency ω of the incoming wave and their harmonics $0\omega, 2\omega, 3\omega, \ldots$. With an asymptotic expansion for small viscosities and amplitudes, both scaled like ε^2, we obtained corrector terms of order 1 and 2 for frequency 0, ω, and 2ω, where the static contribution of order 1 couples with the second order solution for $k = 1$.

Based on joint work with K. Schmidt.

Keywords: acoustic wave propagation, singularly perturbed PDE, asymptotic expansions.

Thin domains with oscillatory boundaries beyond the purely periodic case

Manuel VILLANUEVA-PESQUEIRA

Department of Applied Mathematics, Complutense University of Madrid

We analyze the behavior of solutions to Poisson's equation with Neumann boundary conditions posed in two-dimensional thin domains with oscillatory boundaries. We consider thin domains with the following general estructure

$$R^{\epsilon} = \{(x, y) \in \mathbb{R}^2 : 0 < x < 1; -\epsilon H_{\epsilon}(x) < y < \epsilon G_{\epsilon}(x)\},$$

where $G_{\epsilon}, H_{\epsilon} : \mathbb{R} \longrightarrow \mathbb{R}$ are two bounded positive functions which oscillate as ϵ tends to zero.

We are interested in understanding how the oscillatory boundaries beyond the classical periodic setting in homogenization affect the behavior of the solutions. In particular, we will consider thin domains with locally periodic oscillatory boundaries and thin domains where the top and the bottom boundary present oscillations with different profile and different order of frequency. We will adapt methods originally devised to deal with periodic structures in order to obtain the homogenized limit problem as the thickness tends to zero and to provide some corrector results.

Based on joint work with José M. Arrieta.

Keywords: thin domain, oscillatory boundary, homogenization.

Bending phenomena in thin domains.

Elvira ZAPPALE

Department of Indutrial Engineering, University of Salerno, Italy

Some integral representation results dealing with integral functionals which model energies in thin structures will be given. I will focus on model described through one or two vector fields, possibly unrelated and undergoing different behaviours, i.e. fields which may lie in different functional spaces. Some applications to both bending and constrained homogenization in slender domains will be also provided.

Keywords: Dimensional reduction, bending, integral representation.

Mathematical modelling of bridges

Organizers: Elvise Berchio (Politecnico di Torino, Italy) and Alberto Ferrero (Università del Piemonte Orientale, Italy).

Vortex-induced dynamic instabilities in suspension bridges due to parametric and autoparametric resonance

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Andrea ARENA

Department of Structural Engineering, Sapienza University of Rome

The nonlinear dynamical response of long-span suspension bridges is investigated in the case of vortex-induced vibrations (VIV) generated by the air-flow separation across the deck section. The multiplicative forcing can induce parametric oscillations of the bridge. A continuum bridge model is coupled with the aerodynamic, time-varying, loads representative of the VIV phenomenon to obtain the partial differential equations of motion. By employing the Faedo-Galerkin method, the aeroelastic nonlinear PDEs are reduced to their state-space ordinary-differential form and a direct asymptotic approach featuring the method of multiple scales is used to investigate the dynamic instabilities induced by the parametric-type aerodynamic lift and moment having frequency nearly twice the fundamental torsional or flexural frequency. Moreover, the interaction between a 2:1 internal (autoparametric) resonance between the lowest torsional and flexural frequencies and the parametric external excitation is studied.

Based on joint work with W. Lacarbonara and G. Rega.

Keywords: suspension bridges, perturbation analysis, parametric resonance.

Energy Transfer between Modes in a Nonlinear Beam Equation

Ubertino BATTISTI

Politecnico di Torino

We consider the nonlinear nonlocal beam evolution equation introduced by Woinowsky-Krieger

$$\begin{cases} u_{tt} + u_{xxxx} + \Big[P - \frac{2}{\pi}\,\|u_x\|^2_{L^2(0,\pi)}\Big]u_{xx} = f & x \in (0,\pi)\,,\ t > 0\,, \\ u(0,t) = u(\pi,t) = u_{xx}(0,t) = u_{xx}(\pi,t) = 0 & t > 0\,. \end{cases}$$

We analyze the behavior of solutions of the problem with two active modes, hence we are lead to the system of ODE's

$$\begin{cases} \ddot{w}(t) + m^2(m^2 - P)w(t) + m^2\big(m^2 w(t)^2 + n^2 z(t)^2\big)w(t) = 0\,, \\ \ddot{z}(t) + n^2(n^2 - P)z(t) + n^2\big(m^2 w(t)^2 + n^2 z(t)^2\big)z(t) = 0\ . \end{cases}$$

We investigate whether there is an energy transfer between them. The answer depends on the involved parameters and on the energy of the system. Our results are complemented with several numerical experiments.

A motivation of our research is to give some hints about the nonlinear structural behavior of suspension bridges: it is reasonable to expect that if some instability appears in a simplified model, namely if the deck of the bridge is seen as a beam, then similar instabilities will appear in more sophisticated models.

Based on joint work with Elvise Berchio, Alberto Ferrero and Filippo Gazzola.

Keywords: Beam Equation, Transfer of Energy, Nonlinear Modes.

Shape optimization for the deck of a suspension bridge

Davide BUOSO

Politecnico di Torino

We consider a partially hinged rectangular plate modelling the deck of a suspension bridge. We study how deformations of the shape affect the eigenvalues and compute the shape derivatives. This is needed to investigate the possibility of finding a shape functional able to measure the torsional instability of the plate, which is the major phenomenon leading to the collapse of suspension bridges. We show that a simple functional obeying all the rules coming from theoretical and practical evidences does not exist, and that the functionals available in the literature are not reliable.

Based on a joint work with Elvise Berchio and Filippo Gazzola.

Keywords: suspension bridges, shape optimization, eigenvalues.

Analysis of the steady states of elastically-coupled nonlinear double-beam systems

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Filippo DELL'ORO

Department of Mathematics, Politecnico di Milano

Given $\beta \in \mathbb{R}$ and $\varrho, k > 0$, we analyze an abstract version of the nonlinear stationary model in dimensionless form

$$\begin{cases} u'''' - \big(\beta + \varrho \int_0^1 |u'(s)|^2\, ds\big)u'' + k(u - v) = 0 \\ v'''' - \big(\beta + \varrho \int_0^1 |v'(s)|^2\, ds\big)v'' - k(u - v) = 0 \end{cases}$$

describing the equilibria of an elastically-coupled extensible double-beam system subject to evenly compressive axial loads. Necessary and sufficient conditions in order to have nontrivial solutions are established, and their explicit closed-form expressions are found. In particular, the solutions are shown to exhibit at most three nonvanishing Fourier modes. In spite of the symmetry of the system, nonsymmetric solutions appear, as well as solutions for which the elastic energy fails to be evenly distributed. Such a feature turns out to be of some relevance in the analysis of the longterm dynamics, for it may lead up to nonsymmetric energy exchanges between the two beams, mimicking the transition from vertical to torsional oscillations.

Based on joint work with Claudio Giorgi and Vittorino Pata.

Keywords: Coupled-beams structures, steady states, buckling.

Resonance tongues for the Hill equation with squared Duffing coefficients

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Carlo GASPARETTO

Dipartimento di Matematica del Politecnico di Milano

The stability problem of beams and plates naturally leads to the study of the stability of the trivial solution of

$$\ddot{\xi}(t) + (n^4 + m^2 n^2 y(t)^2)\xi(t) = 0,$$

where $y(t)$ is the solution of a Duffing equation while m, n are integer parameters. In this talk we describe the stability regions for the simplified equation

$$\ddot{\xi}(t) + (\gamma + y(t)^2)\xi(t) = 0,$$

in the parameter (δ, γ)-plane, where δ is the shooting level of the particular Duffing equation

$$\ddot{y}(t) + y(t) + y(t)^3 = 0, \qquad y(0) = \delta, \qquad \dot{y}(0) = 0.$$

Keywords: Duffing equation, Hill equation, stability of beams and plates.

An instability result for suspension bridges

Clelia MARCHIONNA

Dipartimento di Matematica, Politecnico di Milano

An important issue in the modeling of suspension bridges is the transfer of energy from flexural to torsional modes of vibration, which may occur even when the external forces are not taken into account. We consider the fish-bone bridge model introduced by Moore, and subsequently improved by Gazzola and coworkers. This is a two degrees of freedom system of PDEs in which the dynamics of the midline of the deck is coupled with the response of the suspension cables. In our work, this response is supposed to be asymptotically linear under traction, and asymptotically constant when compressed. We study a second order system of two ODEs which arises as a single mode Galerkin projection of the fish-bone model. The two unknown represent flexural and torsional modes of vibration of the deck. We establish a condition under which the flexural motions are unstable provided the energy is sufficiently large.

Based on joint work with Stefano Panizzi.

Keywords: torsional instability, Poincaré map, Hill equation.

Enhanced linearized flutter analysis of suspension bridges

Gianfranco PIANA

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We investigate the dynamic stability of a single-span suspension bridge deck, subjected to a uniform transverse wind, through a linearized continuous model. The deck, modelled as an elastic beam of constant cross-section, inextensible and unshearable though deformable in bending and torsion, is connected to the main cables by a continuous system of rigid suspenders; both primary (i.e., Saint Venant) and secondary (i.e., Vlasov) torsional rigidities are taken into account. The integro-differential equations governing the dynamic equilibrium of the deck are derived at first by considering the steady wind component (mean wind). Two separate solutions are obtained, by the Galerkin method, for the antisymmetric and symmetric oscillations, giving the natural frequencies as functions of the aerodynamic loads. Afterwards, dynamic stability (i.e., flutter) is studied by including the unsteady aerodynamic loads in the equations of motion.

Based on joint work with Al. Carpinteri.

Keywords: bridge aeroelasticity, continuous models, second-order effects.

Static and dynamic analysis of laminated plates using enriched macro elements

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Rita F. RANGO

INIQUI - CONICET - Universidad Nacional de Salta, Salta, Argentina

The development and application of polynomially-enriched plate macro element are presented. In the Trigonometric Shear Deformation Theory framework, shear stresses are vanished at the top and bottom surfaces of the plates so shear correction factors are no longer required. The

element is obtained using Gram-Schmidt orthogonal polynomials as enrichment functions. For taking into account plates of several geometrical shapes, a general straight-sided quadrilateral domain is mapped into a square domain in the computational space using a four-node master plate. A special connectivity matrix is obtained, so that hierarchically enriched global stiffness matrix, mass matrix and loading vector of general laminated plate structures are derived, allowing to study generally coplanar plate assemblies by combining two or more macro elements. This procedure gives a matrix equation of static equilibrium and a matrix-eigenvalue problem that can be solved with optimum efficiency. Results of static and free vibration analysis for symmetric laminated plates of different length-to-thickness ratios, geometrical planforms and several boundary conditions are presented. The accuracy of the formulation is ensured by comparing some numerical examples with those available in the literature. The application of the developed macro elements to the study of stiffening trusses of plates, such as bridges, is also analyzed.

Based on joint work with Stefano Panizzi.

Keywords: plates, composite laminates, macro element, trigonometric shear deformation theory.

Wave Phenomena

Organizer: Willy Dörfler (KIT, Karlsruhe, Germany)

Optimization of 3D-Waveguides using Finite Elements and Transformation Optics

Pascal KRAFT

Department of Mathematics, Karlsruhe Institute of Technology (KIT)

Solving complete 3D-Maxwell problems using Finite Elements is a very expensive computational operation. However, for the evaluation of more simplified models it is a useful tool for applications. We present an approach based on transformation optics to reduce the computational cost of both, individual simulations of wave propagation in a Photonic Wire Bond as well as an optimization scheme built on top of them. On one hand the large condition number of system matrices resulting from the application of Nédélec elements to Maxwell's equations usually prevent the usage of iterative solvers. On the other hand the high number of required degrees of freedom to approximate the oscillatory nature of the solution discourage the usage of direct solvers. A specified preconditioner for GMRES changes convergence rates drastically. The presented scheme also offers substantial improvements concerning repeated usage of the same triangulation and exploitation of similarities in the system matrix when simulating similar physical shapes of waveguides.

Keywords: Finite Elements, Transformation Optics, Shape Optimization.

Hardy space method for waveguide problems

Lothar NANNEN

Institute for Analysis and Scientific Computing, TU Wien

We consider the numerical solution of wave equations in domains with cylindrical waveguides. Standard complex scaling methods fail for such problems if there exist modes with different signs of group and phase velocity. Moreover, in the neighborhood of vanishing phase velocities the discretization error typically degenerates.

We present a frequency independent infinite element method solving both issues. Exponential convergence is shown with respect to the number of unknowns in propagation direction. Numerical results for scattering and resonance problems support this result.

Keywords: waveguide problems, infinite element method, pole condition

On Scattering at 2D Open Waveguides

Julian OTT

Karlsruhe Institute for Technology

Motivated by research on photonic wirebonds, we study junctions of open waveguides in $\mathbb{R}^2$: We model those by the Helmholtz equation

$$\Delta u(x) + (k(x)^2 + i\epsilon)u(x) = f(x),\ x \in \mathbb{R}^2 \text{ with } k(x) = k_\delta \chi_\Omega(x) + k_0,$$

where ϵ is a (small) absorption parameter, $k_\delta, k_0 > 0$ are constants and χ_Ω denotes the characteristic function of Ω. The unbounded domain Ω consists of two unbounded waveguides and a bounded junction, connecting the two waveguides. We present a halfspace matching method which allows us to solve this scattering problem numerically in the exterior of a square. This integral equation method is then coupled with a FE–method in the vicinity of the junction to solve the problem in full $\mathbb{R}^2$.
Furthermore, we will present some numerical examples, where this method has been used to optimize the transmission behaviour of a waveguide junction.

Based on joint work with A.-S. Bonnet ben Dhia, S. Fliss, A. Tonnoir.

Keywords: waveguide, scattering, optimization.

Transparent boundary conditions for inhomogenous exterior domains

Achim SCHÄDLE

Department of Mathematics, Heinrich-Heine University, Germany

In this talk the pole condition approach for deriving transparent boundary conditions is extended to the time-dependent, two-dimensional case with an inhomogenous wave-guide like exterior domain. Nonphysical modes of the solution are identified by the position of poles of the solution's spatial Laplace transform in the complex plane. By requiring that the Laplace transform is analytic on some problem-dependent complex half-plane, i.e belongs to a certain Hardy space, these modes can efficiently be suppressed. The resulting algorithm computes a finite number of coefficients of a series expansion of the Laplace transform, thereby providing an approximation to the exact boundary condition. The resulting error decays super-algebraically with the number of coefficients, so relatively few additional degrees of freedom are sufficient to reduce the error to the level of the discretization error in the interior of the computational domain.

Based on joint work with Daniel Ruprecht and Frank Schmidt

Keywords: transparent boundary conditions, pole condition, wave guide.

On absorbing boundary conditions in a waveguide for acoustic models at low viscosity

Kersten SCHMIDT

Research Center MATHEON *and Institut für Mathematik, Technische Universität Berlin, Germany*

The wave-propagation in infinite wave-guides shall be modelled on truncated domains by absorbing boundary conditions on the artificial boundary Σ. In viscous gases acoustic waves exhibit small boundary layers on the side walls. The effective behaviour can be modelled with impedance boundary conditions of Wentzell type that relate normal and second tangential derivatives with a factor depending on the viscosity ν. We present approximative Dirichlet-to-Neumann absorbing boundary conditions satisfying the Wentzel conditions that guarantee a modelling error of $O(\nu)$.
For the discretization by the finite element method the variational formulation does not provide enough regularity for the Wentzell conditions. On the point, where wall and artificial boundary Σ meet we would need the Dirichlet-to-Neumann map in a point-wise sense. As in the formulation it is a mapping from $H^{\frac{1}{2}}(\Sigma)$ to $H^{-\frac{1}{2}}(\Sigma)$ only, we change the variational formulation on the end-points to guarantee well-posedness with constants independent of ν and optimal error estimates.

Based on joint work with Adrien Semin, TU Berlin.

Keywords: Wave-guide, Absorbing boundary conditions, Impedance condition in viscous gases.

Boundary integral equations and boundary element methods for Maxwell's eigenvalue problems

Gerhard UNGER

Graz University of Technology, Austria

We consider a Galerkin approximation of boundary integral formulations of Maxwell's eigenvalue problem in bounded and unbounded domains. An analysis of the boundary integral formulations and their numerical approximations is given in the framework of eigenvalue problems for holomorphic Fredholm operator-valued functions. We show that Galerkin approximations yield a so-called regular convergent sequence for the underlying operator-valued function of the eigenvalue problem. This allows us to apply general results of the numerical analysis of holomorphic eigenvalue problems which guarantees the convergence of the eigenvalues as well as of the eigenspaces. Numerical examples confirm the theoretical results.

Keywords: Maxwell's eigenvalue problems, boundary integral equations, boundary element method.

Wiener-Hopf Techniques and their Applications

Organizers: G. Mishuris (Aberystwyth University, UK), S.Rogosin (University of Belarus, Minsk, Belarus), M. Dubatovskaya (University of Belarus, Minsk, Belarus).

Wiener-Hopf operators on a totally quasi-ordered discrete Abelian group

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Victor ADUKOV

Department of Mathematical and Functional Analysis, South Ural State University, Russian Federation

Let G be a discrete Abelian group and $\leq$ a total quasi-order on G. This means that the relation $\leq$ is reflexive, transitive, translation-invariant, and for all $g, h \in G$, either $g \leq h$ or $h \leq g$. It is known that G can be totally ordered iff there exists an element of infinite order in G. Here we will suppose that G is a torsion group (the case of a torsion free group has been studied previously). Let G_+ be a cone of nonnegative elements of G. Then $G = G_+ \bigcup G_-^*$, where G_+, G_-^* are semigroups, $0 \in G_+$, and $G_+ \bigcap G_-^* = \emptyset$.

Define a Wiener – Hopf operator T_a in $l^p(G_+)$ by the formula

$$T_a\, x(g) = \sum_{h \in G_+} a(g-h)x(h),\ \ g \in G_+.$$

Here a is an invertible element of the group algebra $l^1(G)$. The Fourier transform $\hat{a}$ of a is referred as the symbol of T_a. In the report we will study T_a with the help of the Wiener – Hopf factorization technics for $\hat{a}$. It is shown that T_a is generalized invertible and the generalized inverse of T_a is found in terms of the factorization of $\hat{a}$. A criterion of one-side invertibility of T_a is obtained. The groups G on which there are nontrivial Fredholm operators T_a are described.

Keywords: totally quasi-ordered discrete Abelian group, Wiener – Hopf operator, generalized inversion.

Wiener-Hopf factorization and $\mathfrak{Q}$-classes of matrix functions

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M. Cristina CÂMARA

Center for Mathematical Analysis, Geometry, and Dynamical Systems, Departamento de Matemática, Instituto Superior Técnico-Universidade de Lisboa,Portugal

We generalize the notion of $\mathfrak{Q}$-classes, which was introduced in the context of Wiener-Hopf factorization of matrix functions. This allows us to use a mainly algebraic approach to obtain several equivalent representations for each class, to study the intersections of $\mathfrak{Q}$-classes and to explore their close connection with certain non-linear scalar Riemann-Hilbert problems. The results are applied to various factorization problems.

Based on joint work with M. Teresa Malheiro.

Keywords: Riemann-Hilbert problems, Toeplitz operators, factorization.

Constructive factorization of Wiener-Hopf operators on Sobolev spaces with symmetry

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Luís CASTRO

CIDMA – Center for Research and Development in Mathematics and Applications, Dep. of Mathematics, University of Aveiro, Portugal

Wiener-Hopf type operators with symmetry appear naturally in boundary value problems for elliptic PDEs in symmetric or symmetrizable domains. They are defined as truncations of translation invariant operators in a scale of Sobolev spaces that are convolutionally similar to subspaces of even or odd functionals. We will consider a class of these operators which is closely related to the Helmholtz equation in a quadrant, where a possible solution is symmetrically extended to a half-plane. We will present a constructive factorization technique for such operators which turns possible the representation of consequent resolvent operators in closed analytic form. Moreover, characterizations of the normal solvability, Fredholm property, generalized (and one-sided) invertibility of those operators are also obtained.

Based on joint work with F.-O. Speck.

Keywords: Wiener-Hopf type operators, factorization, Fredholm property.

UNIQUENESS CONDITION FOR THE (LOCAL) SOLUTION OF THE ELECTROSTATIC MICRO - CANTILEVER BEAM

Lorenzo DI RUVO

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A typical micro electromechanical system (MEMS) actuator is formed by a micro cantilever beam electrode suspended above a conductive substrate and subject to a voltage difference. Due to electrostatic forces the micro-beam deflects toward to the substrate and pulls-in onto the substrate plane at a critical voltage, named pull-in voltage, thus causing a short circuit.

Under the pull-in voltage the micro-beam leaves the principal equilibrium path, which becomes unstable due to the increase of the electrostatic force with the beam deflection, and a kind of snap-through instability occurs. The deflection of an elastic cantilever beam is described by a fourth-order non-linear ODE

$$\begin{cases} u^{(IV)} = \dfrac{\beta}{(1-u)^2}, \\ u(0) = u'(0) = 0, \\ u''(1) = u'''(1) = 0, \end{cases} \qquad 0 \le x \le 1, \quad \beta > 0. \tag{1}$$

Using fixed point Theorem, we prove the local uniqueness of the solution of (1), with respect to L^2 norm.

Based on joint work with Enrico Radi.

Keywords: existence, uniqueness, stability.

Fracture waves in a structured media with a moving boundary.

Nikolai GORBUSHIN

Department of Mathematics, Aberystwyth University, UK

Experimental observations and numerical simulations reveal various phenomena during steady-state fault propagations such as elastic wave radiation from the moving crack tip, various surface roughness depending on the crack speed, etc. Mathematical description of the problem mainly considered as developed still lacks full understanding.

We study the steady-state crack propagation in one-dimensional chain of oscillators with non-local interactions. The oscillators are interconnected by linear springs to the four closest neighbours (two from both sides). We used a technique based on Wiener-Hopf method, introduced by Slepyan L.I. and developed further by him and his colleagues, to study steady-state crack propagation in a structured media.

The obtained results demonstrate that, apart from the basic energetic considerations, a thorough analysis of solution is required for the prediction of the admissible crack motion. Moreover, the variation of parameters of the problem can significantly effect the solutions. Presentation is supported by numerical simulations.

Based on joint work with G. Mishuris.

Keywords: waves, dynamic fracture, Wiener-Hopf technique.

On Wiener-Hopf and Mellin operators arising in the theory of Lévy processes

Tony HILL

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Markov processes are well understood in the case when they take place in the whole Euclidean space. However, the situation becomes much more complicated if a Markov process is restricted to a domain with a boundary, and then a satisfactory theory only exists for processes with continuous trajectories. The aim of this talk is to present interim results of an ongoing research project concerned with a symmetric stable Lévy process defined on a half-line. After some analysis, the problem is reformulated in the context of an algebra of multiplication, Wiener-Hopf and Mellin operators, and the nature and significance of the resulting symbol is examined.

Based on joint work with Eugene Shargorodsky.

Keywords: Wiener-Hopf, Mellin, Lévy process.

Approximate factorisation of matrix Wiener–Hopf arising in acoustics.

Anastasia KISIL

University of Cambridge, Corpus Christi College

I will talk about two matrix Wiener-Hopf problems which result from the application of the Wiener-Hopf technique to acoustics. The first one is motivated by studying the effect of a finite elastic trailing edge on noise production. The approximate factorisation of this matrix with exponential phase factors is achieved using an iterative procedure which makes use of the scalar Wiener-Hopf problem arising for each junction.

The second matrix problem arise from scattering of sound waves by an infinite grating composed of rigid plates. The approximate factorisation of this matrix resulting from a periodic structure is performed using conformal mapping, rational approximation and then the recent procedure by Mishuris & Rogosin.

The first part is based on joint work with Prof. Peake and the second with Prof. Abrahams.

Keywords: matrix Wiener–Hopf, approximate factorisation, acoustics .

Asymptotic Methods for Factorization of a Class of Matrix Functions with Stable Partial Indices

Gennady MISHURIS

Department of Mathematics, Aberystwyth University, UK

A new effective method for factorisation of a class of nonrational $n \times n$ matrix-functions with *stable partial indices* is proposed. The method is a generalization of one recently proposed by the authors, which was valid for the canonical factorisation only. The class of matrices being considered is motivated by their applicability to various problems. The properties and steps of the asymptotic procedure are discussed in detail. The efficiency of the procedure is highlighted by numerical results.

Based on joint work with S. Rogosin.

Keywords: matrix-functions, factorization, stable partial indices.

Weight functions analysis for interfacial cracks in piezoelectric materials

Lorenzo MORINI

IMT School for Advanced Studies, Lucca, Italy

The problem of a semi-infinite plane interfacial crack between two dissimilar piezoelectric materials is reduced to a matrix eigenvalue problem by means of extended Stroh formalism combined with Riemann-Hilbert formulation. The non-trivial singular solutions of the homogeneous problem for the crack are the weight functions. Symmetric and skew-symmetric component of the weight functons are obtained by the analytical solution of the homogeneous equations avoiding the challenging analysis of the matrix functional Wiener-Hopf equation commonly performed for semi-infinite crack problems. The weight functions matrices are then used together with Betti's reciprocity identity in order to formulate the problem in terms of singular integral equations relating the extended displacement and traction fields to the loading acting on the crack faces.

Based on joint work with D. Andreeva, L. Pryce and Prof. G. Mishuris.

Keywords: Piezoelectric bimaterial, Weight functions, Riemann-Hilbert formulation.

Steady state propagation of a rectilinear crack in a thin elastic plate supported by a Winkler elastic foundation

Andrea NOBILI

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We consider steady state propagation of a rectilinear crack in a infinite thin elastic Kirchhoff plate bilaterally supported by an elastic Winkler foundation. The crack flanks are subjected to a continuous (between the flanks) harmonic load. The problem's governing equation features the biharmonic operator together with a curvature (along the crack) term. Through application of the Fourier transforms to the even/odd part of the problem, a pair of inhomogenenous uncoupled Weiner-Hopf equations is met. Solution is obtained through numeric factorization of the kernel function. The full-field solution is given, together with conditions on the energy radiation. The special case of stationary crack is also retrieved.

Based on joint work with Enrico Radi, Luca Lanzoni.

Keywords: Crack propagation, Kirchhoff plate, Weiner-Hopf equations.

Constructive Methods for Factorization of Matrix Functions

Sergei ROGOSIN

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A survey of constructive methods for the factorization of $n \times n$ matrix functions is presented. The importance of these methods for theoretical and practical applications is singled out. Several classes of matrices are considered which are factorized by the proper technique. The perspective of the constructive methods and procedures is discussed and open questions are formulated. Partly the obtained results are published in IMA Journal of Applied Mathematics.

Based on joint work with G. Mishuris and M. Dubatovskaya.

Keywords: matrix-functions, factorization, constructive methods.

On potential methods for porous media flows

Wolfgang WENDLAND

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We obtain existence and uniqueness results in Sobolev spaces for transmission problems for the nonlinear Darcy–Forchheimer–Brinkman system in a Lipschitz domain and the linear Stokes system in the exterior in $\mathbb{R}^3$ and also on compact Riemannian manifolds. We apply a layer potential method for the Stokes and Brinkman systems combined with a fixed point argument to obtain existence. Uniqueness is also obtained for sufficiently small given data.

Based on joint work with Mirela Kohr (Babeş Bolyai Univ., Cluj–Napoca), Massimo Lanza de Cristoforis (Univ. Padua) and Sergey E. Mikhailov (Brunel University, Uxbridge).

Keywords: transmission problem, Brinkman system, layer potential.