Interaction models: Mean Field Games, pattern formation and related topics

January 25-26, 2018

Abstracts

Stable solutions in potential mean field games

Pierre Cardaliaguet
Université Paris-Dauphine

In this joint work with Ariela Briani, we introduce the notion of stable solution of a mean field game system. We prove that such solutions exist in potential mean field games and are local attractors for some learning procedures.

Concentration phenomena for Mean Field Games systems with aggregation

Annalisa Cesaroni
Università di Padova

I will present some results obtained in collaboration with Marco Cirant about stationary mean field game systems, describing equilibrium configurations for a mass population of rational individuals each of whom subjected to a coercive potential and an aggregation force. I will describe the variational framework for the analysis of these problems, and discuss concentration phenomena arising in the semiclassical limit.

Long time behavior of mean-field games

Alessio Porretta
Università di Roma Tor Vergata

In this talk I will discuss the long time behavior of mean-field games systems in the stable case (monotone couplings) when the dynamic takes place in the flat torus. I will present the main features that appeared in the study of the long time limit: the effects of the forward-backward coupling, the ergodic behavior and the turnpike property of the underlying control problems, the long time description of the master equation. Joint works with P. Cardaliaguet, J-M. Lasry and P.-L. Lions.

On the variational formulation of some stationary second order MFGs

Francisco Silva

Université de Limoges

In this talk we consider an extension of the work by A. Mészàros and myself (15') dealing with variational stationary MFGs with density constraints. We consider general Hamiltonians, satisfying a suitable growth condition, and also rather general coupling terms. For systems with and without density constraints we establish the existence of solutions using a variational technique and we also improve some of our previous results.

Variational problems with long-range interactions

Nicola Soave

Politecnico di Milano

We consider a class of variational problems for densities that repel each other at distance. Typical examples are given by the Dirichlet functional and the Rayleigh quotient

$$D(\mathbf{u}) = \sum_{i=1}^{k} \int_{\Omega} |\nabla u_i|^2 \quad \text{and} \quad R(\mathbf{u}) = \sum_{i=1}^{k} \frac{\int_{\Omega} |\nabla u_i|^2}{\int_{\Omega} u_i^2},$$

minimized in the class of $H^1(\Omega, \mathbb{R}^k)$ functions attaining some boundary conditions on $\partial\Omega$, and subject to the constraint

$$\operatorname{dist}(\{u_i > 0\}, \{u_j > 0\}) \geq 1 \qquad \forall i \neq j.$$

For these problems, we investigate the optimal regularity of the solutions, prove a free-boundary condition, and derive some preliminary results characterizing the free boundary $\partial\{\sum_{i=1}^{k} u_i > 0\}$. This is a joint work with H. Tavares, S. Terracini and A. Zilio.

Semitrivial and fully nontrivial solutions for cubic Schrödinger systems

Hugo Tavares
Universidade de Lisboa

In this talk we focus on the cubic Schrödinger system with $d \geq 3$ equations:

$$-\Delta u_i + \lambda_i u_i = \mu_i u_i^3 + u_i \sum_{j \neq i} \beta_{ij} u_j^2 \quad \text{in} \Omega \subseteq \mathbb{R}^N, \qquad i = 1, ...d,$$

where $\lambda_i, \mu_i > 0$, $\beta_{ij} = \beta_{ji} \in \mathbb{R}$ for $j \neq i$, $N = 2, 3$. The underlying domain Ω is either bounded or the whole space, and $u_i \in H_0^1(\Omega)$ or $u_i \in H_{rad}^1(\mathbb{R}^N)$ respectively. For the case $\beta_{ij} > 0$ (called cooperative), we provide optimal qualitative conditions on the parameters λ_i, mu_i and β_{ij} under which the ground state solutions have all components nontrivial, or, conversely, are semitrivial. Moreover, in the presence of both cooperation and competition coefficients, we present existence and symmetry results of positive solutions. Joint works with S. Correia, F. Oliveira, N. Soave

---

Spiralling and other solutions in limiting profiles of competition-diffusion systems

Susanna Terracini
Università di Torino

Several physical phenomena can be described by a certain number of densities (of mass, population, probability, ...) distributed in a domain and subject to laws of diffusion, reaction, and *competitive interaction*. Whenever the competitive interaction is the prevailing phenomenon, the several densities can not coexist and tend to segregate, hence determining a partition of the domain *(Gause's experimental principle of competitive exclusion (1932))*. As a model problem, we consider the system of stationary equations:

$$\begin{cases} -\Delta u_i = f_i(u_i) - \beta u_i \sum_{j \neq i} g_{ij}(u_j) \\ u_i > 0 \ . \end{cases}$$

The cases $g_{ij}(s) = \beta_{ij} s$ (Lotka-Volterra competitive interactions) and $g_{ij}(s) = \beta_{ij} s^2$ (gradient system for Gross-Pitaevskii energies) are of particular interest in the applications to population dynamics and theoretical physics respectively.

We will undertake the analysis of qualitative properties of solutions to systems of semilinear elliptic equations, whenever the parameter β, accounting for the competitive interactions, diverges to infinity. At the limit, when the minimal interspecific competition rate $\beta = \min_{ij} \beta_{ij}$ diverges to infinity, we find a vector $U = (u_1, \cdots, u_h)$ of functions with mutually disjoint supports: *the segregated states*: $u_i \cdot u_j \equiv 0$, for $i \neq j$, satisfying

$$-\Delta u_i = f_i(x, u_i) \qquad \text{whenever } u_i \neq 0 \ , \quad i = 1, \ldots, h,$$

We will review the known results and focus on spiralling solutions in the non symmetrical case: ($\beta_{ij} \neq \beta_{ji}$).

On the planning problem in Mean Field Games systems

Daniela Tonon
Université Paris-Dauphine

In this talk we consider a Mean Field Games system where the initial and final densities of the population are prescribed. This problem has been shown firstly by Lions as a planning model for which ones would like that, when the agents follow their optimal strategy, their density evolves from an initial configuration to a final one at a fixed time horizon. Very few results exist on this problem. The main idea of this talk is to show the existence and uniqueness of weak solutions when the initial and final configurations are some given prescribed measures (not necessarily regular) by using variational techniques coming from optimal transport introduced by Benamou and Brenier, extended by Carlier, Cardaliaguet and Nazaret, and which were already used to show the existence and uniqueness of weak solutions for standard first order and second order degenerate MFGs by Cardaliaguet and collaborators. This work is a joint ongoing collaboration with F.J. Silva.

Predators-prey model with competition, the emergence of packs and territoriality

Alessandro Zilio
Université Paris Diderot

I will present a series of works in collaboration with Henri Berestycki, dealing with systems of predators interacting with a single prey. The system is linked to the Lotka-Volterra model of interaction with diffusion, but unlike more classic works, we are interested in studying the case where competition between predators is very strong: in this context, the original domain is partitioned in different sub-territories occupied by different predators. The question that we ask is under which conditions the predators segregate in packs and whether there is a benefit to the hostility between the packs. More specifically, we study the stationary states of the system, the stability of the solutions and the bifurcation diagram, and the asymptotic properties of the system when the intensity of the competition becomes infinite.
