Robust linear quadratic mean-field games in crowd-seeking social networks

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Padua, 5 Sept. 2013

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Why we need more property taxes
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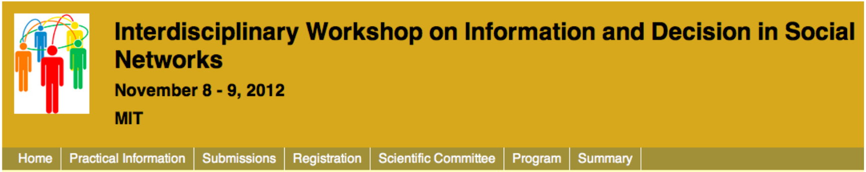

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opinion dynamics

SIAM J. CONTROL OPTIM.
Vol. 48, No. 8, pp. 5214–5240

CONTINUOUS-TIME AVERAGE-PRESERVING OPINION DYNAMICS WITH OPINION-DEPENDENT COMMUNICATIONS*

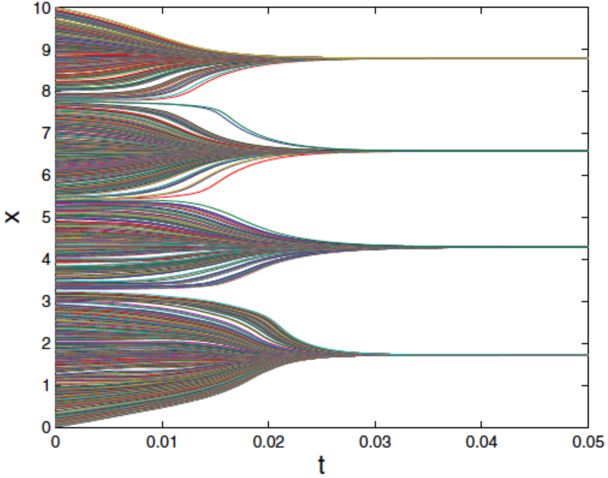
VINCENT D. BLONDEL†, JULIEN M. HENDRICKX†, AND JOHN N. TSITSIKLIS‡

Abstract. We study a simple continuous-time multiagent system related to Krause's model of opinion dynamics: each agent holds a real value, and this value is continuously attracted by every other value differing from it by less than 1, with an intensity proportional to the difference. We prove convergence to a set of clusters, with the agents in each cluster sharing a common value, and provide a lower bound on the distance between clusters at a stable equilibrium, under a suitable notion of multiagent system stability. To better understand the behavior of the system for a large number of agents, we introduce a variant involving a continuum of agents. We prove, under some conditions, the existence of a solution to the system dynamics, convergence to clusters, and a nontrivial lower bound on the distance between clusters. Finally, we establish that the continuum model accurately represents the asymptotic behavior of a system with a finite but large number of agents.

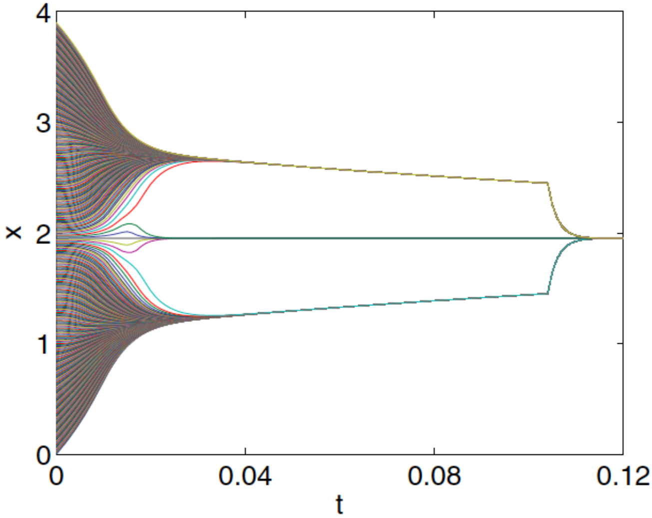
Key words. multiagent systems, consensus, opinion dynamics

AMS subject classifications. 93A14, 91C20, 91D99, 45K05, 45G10, 37F99

Clusters



Stability



Hegselmann-Krause model

Journal of Artifical Societies and Social Simulation (JASSS) vol.5, no. 3, 2002
http://jasss.soc.surrey.ac.uk/5/3/2.html

OPINION DYNAMICS AND BOUNDED CONFIDENCE MODELS, ANALYSIS, AND SIMULATION*

Rainer Hegselmann
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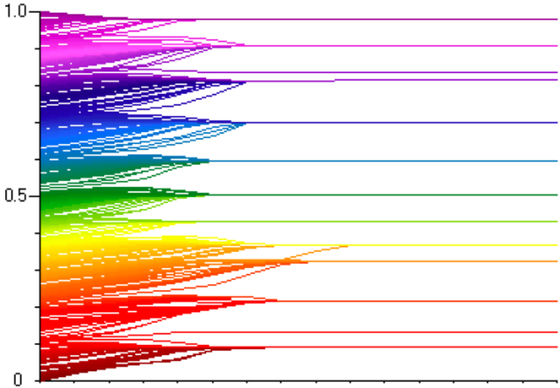
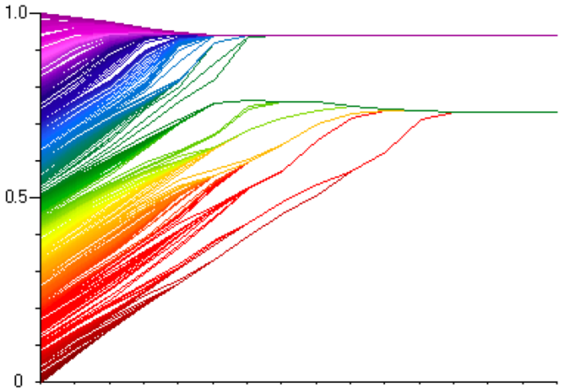
Ulrich Krause
Department of Mathematics, University Bremen

Abstract When does opinion formation within an interacting group lead to consensus, polarization or fragmentation? The article investigates various models for the dynamics of continuous opinions by analytical methods as well as by computer simulations. Section 2 develops within a unified framework the classical model of consensus formation, the variant of this model due to Friedkin and Johnsen, a time-dependent version and a nonlinear version with bounded confidence of the agents. Section 3 presents for all these models major analytical results. Section 4 gives an extensive exploration of the nonlinear model with bounded confidence by a series of computer simulations. An appendix supplies needed mathematical definitions, tools, and theorems.

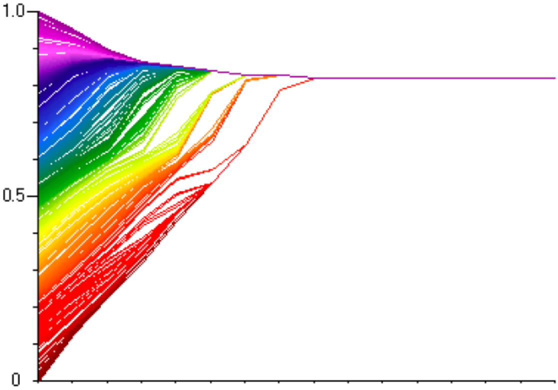
plurality



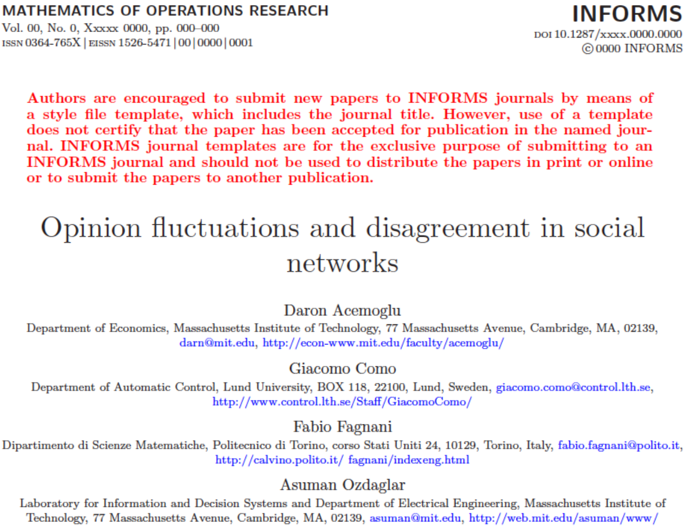
polarization



consensus



random matching and stubbornness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Opinion fluctuations and disagreement in social networks

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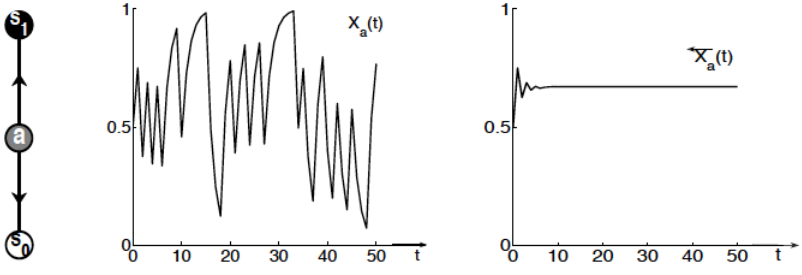
convergence in probability



(a) (b) (c)

How can MFG theory help?

Submitted to *Operations Research*
manuscript (Please, provide the manuscript number!)

Opinion dynamics in social networks through mean field games

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Stubborn agents as disturbances

stubborn

$$\begin{cases} dX(t) = f(X(t), u(t), w(t))dt + \sigma d\mathcal{B}(t), \ t > 0 \\ X(0) = x, \end{cases}$$

- $X : [0,T] \to \mathbb{R}^n$, $t \mapsto X(t)$, group's opinion at time t,
- x its initial opinion at time $t = 0$;
- $u : [0,T] \to U$, $t \mapsto u(t)$, control at time t;
- $w : [0,T] \to W$, $t \mapsto w(t)$, disturbance at time t;
- $\mathcal{B}(t)$ is a standard vector Brownian motion

averaging opinions

- running cost

$$g(x,\overline{m},u,w) = \frac{1}{2}\left[(\overline{m}-x)^T Q\,(\overline{m}-x) + u^T C u + w^T \Gamma w\right]$$

where $Q > 0$, $C > 0$, $\Gamma < 0$ and all symmetric.

- $\overline{m}$ is average opinion
- terminal cost

$$\Psi(x,\overline{m}) = \frac{1}{2}(\overline{m}-x)^T S(\overline{m}-x)$$

where $S > 0$

- cost functional

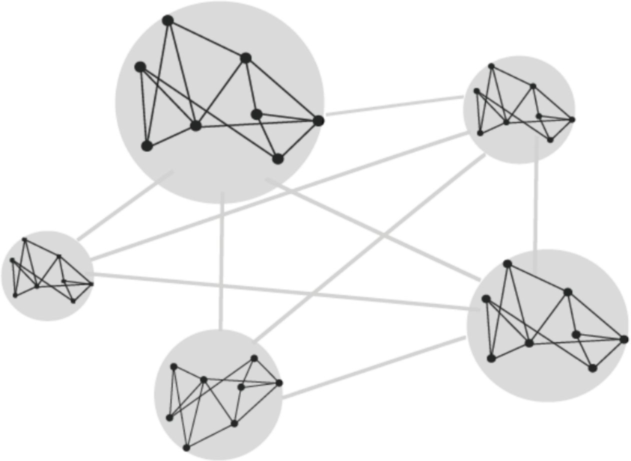
$$\inf_{u(\cdot)\in\mathcal{U}} \sup_{w(\cdot)\in\mathcal{W}} \mathbb{E}\Big[\int_0^T g(X(t),\overline{m}(t),u(t),w(t))dt + \Psi(X(T),\overline{m}(T))\Big]$$

Robust mean field game

$$
\begin{cases}
\partial_t v(x,t) + \inf_{u \in U} \sup_{w \in W} \{f(x,u,w)\partial_x v(x,t) + g(x,\overline{m},u,w)\} \\
+\frac{\sigma^2}{2} Tr\Big(\partial^2_{xx} v(x,t)\Big) = 0 \text{ in } \mathbb{R}^n \times [0,T[, \\
\\
v(x,T) = \Psi(x,\overline{m}) \ \forall \ x \in \mathbb{R}^n, \\
\\
\partial_t m(x,t) + div(m(x,t) \cdot f(x,u^*,w^*)) - \frac{\sigma^2}{2} Tr(\partial^2_{xx} m(x,t)) = 0, \\
\\
m(0) = m_0,
\end{cases}
$$

$$
\begin{cases}
u^*(t,x) \in \arg\min_{u \in U} \{f(x,u,w^*)\partial_x v(x,t) + g(x,\overline{m},u,w^*)\}, \\
\\
w^*(t,x,u) \in \arg\max_{w \in W} \{f(x,u,w)\partial_x v(x,t) + g(x,\overline{m},u,w)\}.
\end{cases}
$$

Network of (homogeneous) networks



Politopic bounds

- control set $U = \Delta(\mathbb{R}^p)$, ($\Delta(\mathbb{R}^p)$ is simplex in $\mathbb{R}^p$, $p > 0$)
- disturbance set $W = \Delta(\mathbb{R}^q)$, $q > 0$
- function $f : U \times W \to [-1, 1]^n$ is bilinear
- function $g : \mathbb{R}^n \times \mathbb{R}^n \to [0, +\infty[$ independent of u, w

$$\begin{cases} \inf_{u(\cdot)\in\mathcal{U}} \sup_{w(\cdot)\in\mathcal{W}} \mathbb{E}\Big[\int_0^T g(X(t), \overline{m}(t))dt + \Psi(X(T), \overline{m}(T))\Big] \\ \\ dX(t) = f(u(t), w(t))dt + \sigma d\mathcal{B}(t), \quad X(0) = x. \end{cases}$$

$\mathcal{L}_2$-gain bounds

- the control set $U = \mathbb{R}^p$, $p > 0$,
- the disturbance set $W = \mathbb{R}^q$, $q > 0$,
- function $f : U \times W \to \mathbb{R}^n$ is linear and of the form,

$$f(u, w) = Fu + Ew,$$

where $F \in \{-1, 0, 1\}^{n \times p}$ and $E \in \{-1, 0, 1\}^{n \times q}$
- the running cost

$$g(x, \overline{m}, u, w) \quad = \quad \frac{1}{2}\left[(\overline{m} - x)^T Q\,(\overline{m} - x) + u^T C u - \gamma^2 w^T w\right],$$

$$\left\{\begin{array}{l} \inf_{u(\cdot)\in\mathcal{U}} \sup_{w(\cdot)\in\mathcal{W}} \mathbb{E}\Big[\int_0^T \frac{1}{2}[(\overline{m} - X(t))^T Q\,(\overline{m} - X(t)) \\ +u(t)^T C u(t) - \gamma^2 w(t)^T w(t)]dt + \Psi(X(T), \overline{m}(T))\Big]\Big\}, \\ \\ dX(t) = Fu(t) + Ew(t)dt + \sigma dB(t), \quad X(0) = x. \end{array}\right.$$

consensus error stochastically bounded

- $e_i(t) :=$ "average opinion" - "ith opinion"
- For each $\pi > 0$ there exists an $\varepsilon(\pi) > 0$ such that

$$\mathbb{P}(\|e_i(t)\|_\infty \leq \varepsilon(\pi)) > 1 - \pi \qquad (1).$$

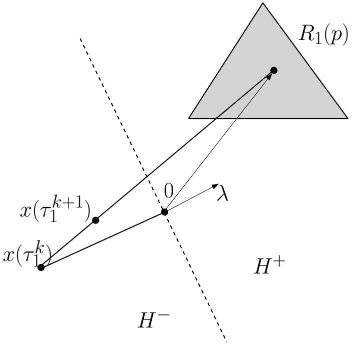
- **Necessary and sufficient condition for (1):**

$$val[\lambda] := \inf_{u \in \Delta(U)} \sup_{w \in \Delta(W)} \{\langle f(u,w), \lambda \rangle\} > 0, \quad \forall \lambda \in \mathbb{R}^n.$$

value of projected game

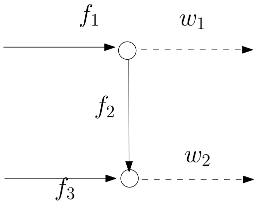
- **Necessary and sufficient condition for (1):**

$$val[\lambda] := \inf_{u\in\Delta(U)}\sup_{w\in\Delta(W)}\{\langle f(u,w),\lambda\rangle\} > 0, \quad \forall\, \lambda \in \mathbb{R}^n.$$



$$\begin{pmatrix} (\#,\#) & (\#,\#) \\ (\#,\#) & (\#,\#) \end{pmatrix} \Rightarrow \begin{pmatrix} \langle\lambda,(\#,\#)\rangle & \langle\lambda,(\#,\#)\rangle \\ \langle\lambda,(\#,\#)\rangle & \langle\lambda,(\#,\#)\rangle \end{pmatrix}$$

Example



$$u(t) \in \left\{ \begin{bmatrix} 1 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -5 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ 6 \end{bmatrix} \right\}$$

$$w(t) \in \left\{ \begin{bmatrix} -3 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \end{bmatrix} - \begin{bmatrix} w_1(t) \\ w_2(t) \end{bmatrix}$$

$$\begin{pmatrix} (6,7) & (1,7) & (6,2) & (1,2) \\ (6,-4) & (1,-4) & (6,-9) & (1,-9) \\ (-3,-1) & (-8,-1) & (-3,-6) & (-8,-6) \\ (-3,10) & (-8,10) & (-3,5) & (-8,5) \end{pmatrix}$$

Monte Carlo simulations

resulting MFG

$$
\begin{cases}
\partial_t v(x,t) + \|\partial_x v\| val[\partial_x v] + \frac{1}{2}(\overline{m}(t)-x)^T Q(\overline{m}(t)-x) \\
+\frac{1}{2}\sigma^2 Tr(\partial^2_{xx} v(x,t)) = 0, \quad \text{in } \mathbb{R}^n \times [0,T[, \\
\\
v(x,T) = \Psi(\overline{m}(T), x), \ \text{in } \mathbb{R}^n, \\
\\
\partial_t m(x,t) + div(m(x,t)\cdot A_{i^* j^*}) - \frac{\sigma^2}{2} Tr(\partial^2_{xx} m) = 0, \ \text{in } \mathbb{R}^n \times [0,T[, \\
\\
m(x,0) = m_0(x) \ \text{in } \mathbb{R}^n.
\end{cases}
$$

$$
\begin{cases}
u^*(x,t) = i^* = \arg\min_{i\in I} \sup_{j\in J} \lambda(\partial_x v)^T A_{ij} \\
w^*(x,t) = j^* = \arg\max_{j\in J} \lambda(\partial_x v)^T A_{ij}.
\end{cases}
$$

$\mathcal{L}_2$-gain bounds

- the control set $U = \mathbb{R}^p$, $p > 0$,
- the disturbance set $W = \mathbb{R}^q$, $q > 0$,
- function $f : U \times W \to \mathbb{R}^n$ is linear and of the form,

$$f(u, w) = Fu + Ew,$$

where $F \in \{-1, 0, 1\}^{n\times p}$ and $E \in \{-1, 0, 1\}^{n\times q}$
- the running cost

$$g(x, \overline{m}, u, w) \;=\; \frac{1}{2}\left[(\overline{m} - x)^T Q\,(\overline{m} - x) + u^T C u - \gamma^2 w^T w\right],$$

$$\begin{cases} \inf_{u(\cdot)\in\mathcal{U}} \sup_{w(\cdot)\in\mathcal{W}} \mathbb{E}\Big[\int_0^T \frac{1}{2}[(\overline{m} - X(t))^T Q\,(\overline{m} - X(t)) \\ +u(t)^T C u(t) - \gamma^2 w(t)^T w(t)]dt + \Psi(X(T), \overline{m}(T))\Big]\Big\}, \\ \\ dX(t) = Fu(t) + Ew(t)dt + \sigma dB(t), \quad X(0) = x. \end{cases}$$

consensus error stochastically bounded

- $e_i(t) :=$ "average opinion" - "ith opinion"
- For each $\pi > 0$ there exists an $\varepsilon(\pi) > 0$ such that

$$\mathbb{P}(\|e_i(t)\|_\infty \leq \varepsilon(\pi)) > 1 - \pi. \tag{1}$$

- **Necessary and sufficient condition for (1):**

$$-FC^{-1}F^T + \frac{1}{\gamma^2}EE^T \leq 0$$

- interpretation (if $C = I$): relation between *algebraic connectivity* of the Laplacian matrix FF^T and the maximal eigenvalue of the Laplacian matrix EE^T.

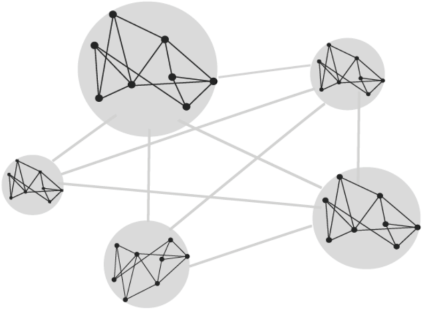
resulting MFG

$$
\begin{cases}
\partial_t v(x,t) + \frac{1}{2}\partial_x v(x,t)^T\Big(-FC^{-1}F^T + \frac{1}{\gamma^2}EE^T\Big)\partial_x v(x,t) \\
+\frac{1}{2}(\overline{m}(t)-x)^T Q(\overline{m}(t)-x) + \frac{1}{2}\sigma^2 Tr(\partial^2_{xx} v(x,t)) = 0, \\
\quad \text{in } \mathbb{R}^n \times [0,T[, \\
\\
v(x,T) = \Psi(\overline{m}(T),x), \ \text{in } \mathbb{R}^n, \\
\\
\partial_t m(x,t) + div\Big(m(x,t)(-\frac{1}{2}C^{-1}FF^T + \frac{1}{2\gamma^2}EE^T)\partial_x v(x,t)\Big) \\
-\frac{1}{2}\sigma^2 Tr(\partial^2_{xx} m(x,t)) = 0, \ \text{in } \mathbb{R}^n \times [0,T[, \\
\\
m(x,0) = m_0(x) \text{ in } \mathbb{R}^n.
\end{cases}
$$

$$
\begin{cases}
u^*(x,t) = -C^{-1}F^T\partial_x v(x,t) \\
w^*(x,t) = \frac{1}{\gamma^2}E^T\partial_x v(x,t).
\end{cases}
$$

Conclusions

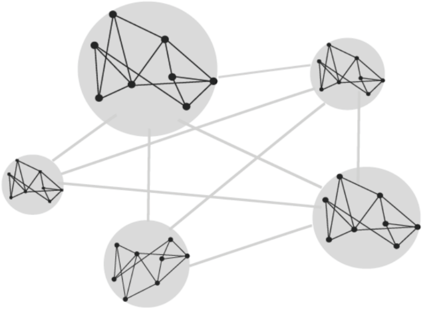
- Network of networks



example of fancy network

Conclusions

- Network of networks and “disturbed” opinion dynamics



- Mean field game formulation for politopic and $\mathcal{L}_2$ bounds
- Stochastic boundedness

No man is an island

No man is an Iland, intire of itselfe; every man is a peece of the Continent, a part of the maine ...

MEDITATION XVII Devotions upon Emergent Occasions John Donne

Questions?

Thank you!