Time to build :

mfg monotone systems approach

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Jean Michel Lasry, 6 septembre 2013
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from a joint work with
Pierre Noël Giraud et Pierre Louis Lions

Time to build

- Old topic in economics :
  - « Time to build and aggregate fluctuations », Kyland and Prescott, Econometrica, 1982
- MFG monotone systems provide a new mathematical tool to investigate this topic
- aim : detail this claim through a « toy » model

agenda

Industry of power plants : modelling stylized dynamics :

- a) One type of power plants (didactic case)
- b) Two different types of power plants
- c) Impact of tax and other non market features on methodology : MFG vs BP optimisation

- *We focus on the industry of power plants as an example of an industry for which « time to build » is a serious issue*
- *Of course, this « toy » model is more generic, i-e: this modelling framework can be applied to many other industries.*

Model with identical power plants

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Stationary equilibrium : value function

- Focus on stationary equilibrium
- Value function u(x) = discounted pay off per one unit of capacity of production
- x is the production capacity = number of production units
- Agents (owners of production units) are atomized independent competitors with constant discounting rate r.

Flow of entrants : the time to build issue

- q^* is the flow of entrants = number of new units
- Cost of a new units is exogenous : $C(q)$
- C is the cost function of the industry which produce new power plants
- Competitive equilibrium of entrants :
- $C'(q^*)= u(x)$
- Ex: $C(q) = \frac{1}{2} q2$ for $q>0$, hence $q^*= u(t,x)$ for $u>0$
- Convexity of the cost function C embodies (in this model) the « time to build » issue

Dynamics of the production capacity

- The dynamics of x is

$$dx = (q^* - a)\,dt$$

where $q^*=\max(u,0)$ is the flow of entrants

and a is a constant aging rate

Demand and pay off

- The demand is exogenous : y=D(p), where y is the production and p the price
- Example :
  - D(p)= 1/pa
  - D(p) = b-cp   for p < b/c
- Constant cost e per unit of produced energy
- Power plants are identical, demand/offer competitive equilibrium : x = D(p)
- Pay off per one production unit :   D-1(x) - e

Recursive (mfg) equation

- We look for a stationary equilibrium:
- u(x) = (1-rdt-adt) u(x+dx) + (D-1(x) – e) dt
- dx = (q*-a) dt , with q*= max(u,0)

$$0 = -\ c\,u + g(u)\,ux + f(x)$$

$$g(u) = \max(u,0) - a; \quad f(x) = D\text{-}1(x) - e; \quad c = r + a$$

- g and –f are increasing functions

HJB and BP

- Define G,F and U has primitives of g,f and u :

  (i-e : $G' = g$, $F' = f$, $U_x = u$)
- then U satisfies the HJB equation:

$$0 = - c\,U + G(U_x) + F(x)$$

- Hence U is the Bellman value function of the control problem :

MFG vs BP

- This means that in this case, the MFG equilibrium is identical to a Benevolent Planner optimisation problem
- We will come back later on this important point
- The BP optimisation problem is not the Monopolist optimisation problem

MFG - BP / Monopolist example

Time to build:
Industry dynamics with two types of power plants and two

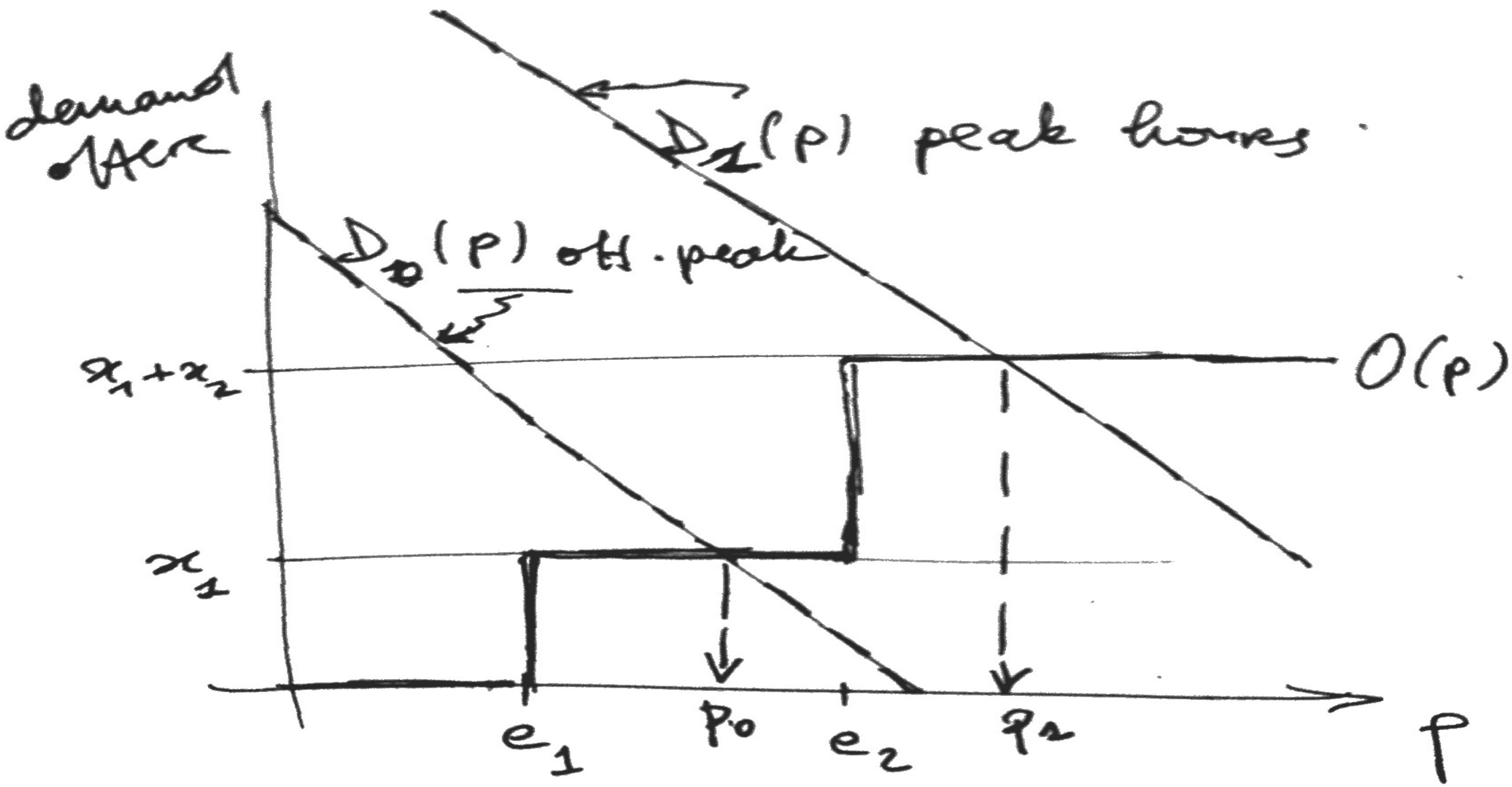
oveview (1/3)

- Two types of power plants
  - Type 1: expensive to build, produce unexpensive energy
  - Type 2: unexpensive to build, produce expensive energy
- Two markets for energy
  - Peak hours: high demand for energy, both unexpensive and expensive energy can be sold
  - Off peak hours: low demand for energy, only unexpensive energy can be sold

oveview (2/3)

- Type 2 power plants
  - Receive only earnings from peak hours market,
  - but are less expensive to build
- Model will tackle interaction of :
  - Time to build with
  - Competition of two populations of producers on two markets

oveview (3/3)



Value functions

- State of the world in this model is x=(x1,x2) where is xi is the existing number of units of type i
- The values functions u1(x1,x2)   and u2(x1,x2) are defined as (expected) discounted pay off for the owner of one unit of type i
- We look for a stationnary competitive equilibrium, i-e: producers are price takers

Two flows of entrants

- For i=1,2,   qi = flow of entrants of type i = number of new units of type i
- Ci(qi) = cost to build one new unit of type i
- Convexity of Ci will express the « time to build issue » in this model
- Cost to build Ci(q) is assumed to be greater for type 1 units : C1(q) > C2(q)
- Cost to produce one unit of energy ei is greater for type 2 units : e1 < e2
- (NB = notations imply a adequate choice of units)

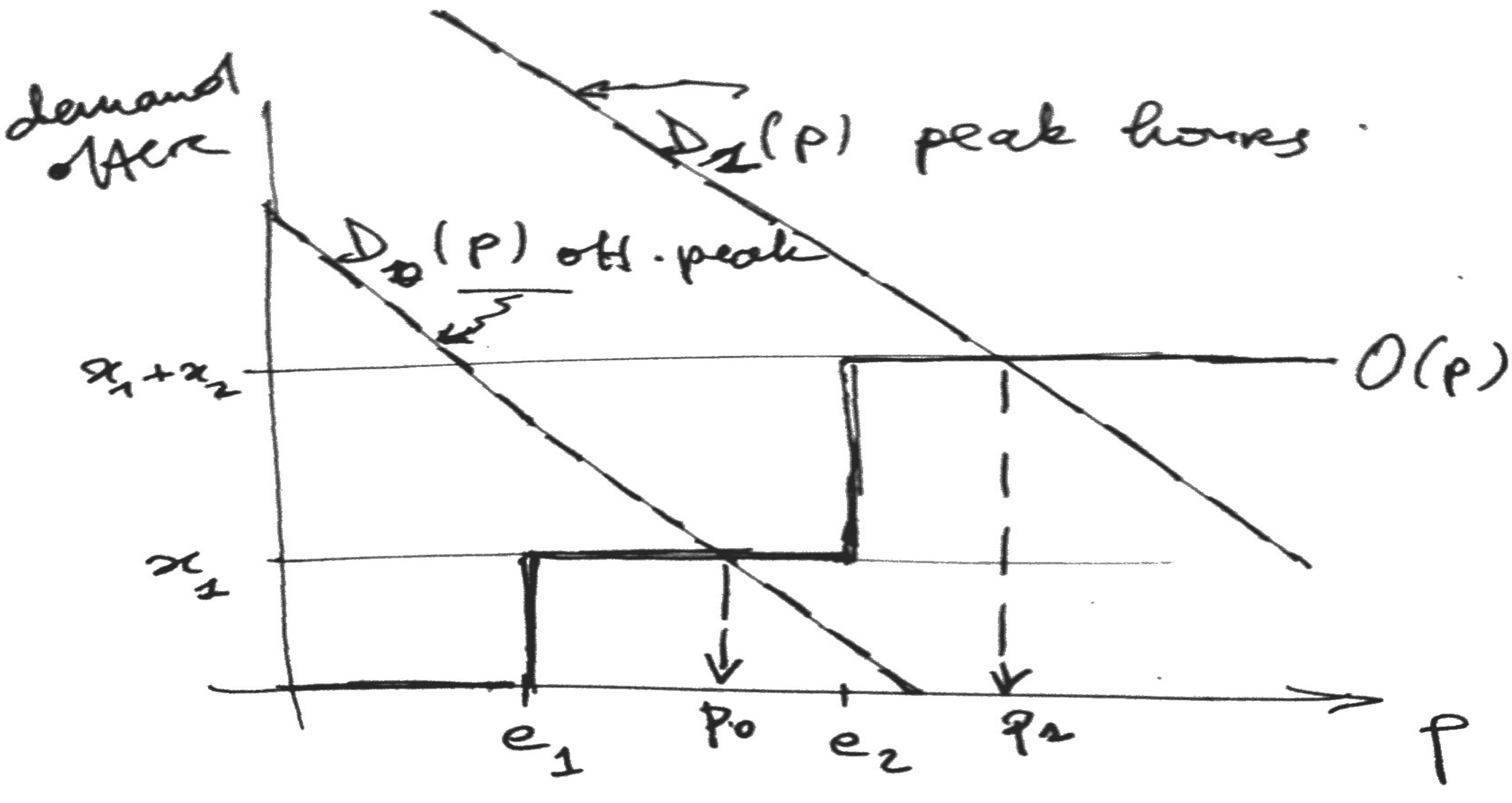
Two flows of entrants

- For the sake of analytical simplicity, we assume $C(q) = \frac{1}{2} c_i q^2$ for $q>0$
- $C_1(q) > C_2(q)$, hence : $c_1 > c_2$
- At equilibrium entrants flows satifies
  - $C_i'(q_i^*) = u_i(x_1,x_2) \quad i=1,2$
  - $c_i q_i^* = u_i(x_1,x_2) \quad i=1,2$

Peak and off-peak demand

- $D_j(p_j)$ is the demand function and p_j the energy price
  - Off peak hours $j=0$
  - Peak hours $j=1$
- Linear case : $D_j(p) = a_j - b_j\, p$, for $p < a_j/b_j$
- Example
  - $a_0 \ll a_1$ $b_0 = b_1$
  - hence off-peak demand D_0 lower than peak demand D_1

Demand and offer
peak and off-peak equilibria



Pay off

- The net pay off for one unit of type 2 is $f2(x1,x2)=p1(x1,x2)-e2$ as this unit produce only for the peak hours market
- The pay off for one unit of type 1 is $f2(x1,x2)=p0(x1,x2)+p1(x1,x2)-e1$ as this unit produces both for the peak and off-peak market
- For the sake of simplicity, we will restrict (here) to the case $e1 < p0 < e2 < p1$
- Hence $p0(x1,x2)=p0(x1)$ and $p1(x1,x2) = p1(x1+x2)$
- More specificaly : $p0=a0-x1$ and $p1= a1-(x1+x2)$

The stationary equilibrium equations

- $ui(x1,x2) = (1-rdt-kdt)\ ui(x1+dx1,x2+dx2) + fi(x1,x2)dt$
- $dxi = gi(x1,x2)\ dt$, with
- $gi(u1,u2) = qi^{*}-k = ui(x1,x2)/ci - k$

(where k is the rate of aging of all units)

Hence :

The previous MFG monotone system, is the « gradient » of an

MFG vs « HJB BP » (1/2)

- Lucas Prescott : any competitive market equilibrium can be framed as the optimal control problem of some (adequatly defined) Benevolent Planer; hence solution proceed in two steps : found the right BP; one can solve the optimization problem of this BP using all classic tools of optimal control (Euler-Lagrange; HJB;..)
- The previous slide illustrate strongly the Lucas Prescott viewpoint : when the MFG equilibrium is a competitive market equilibrium, then the MFG system is the « gradient » of an HJB equation, i-e the HJB equation of the BP optimisation problem

MFG vs « HJB BP » (2/2)

- This defines more clearly the role of MFG systems :
- As soon as there are « non market interactions » between agents:
  - The equivalent BP optimization problem does not exist anymore
  - Then the MFG system become inescapable
- « non market interactions » might be:
  - tax,
  - frictions,
  - externalities,..

Ex. : Tax impact on equilibrium

Remark: analytic solution in the linear case

A glance to agregate shocks