MEAN FIELD GAMES AND RELATED TOPICS 5
BOOK OF ABSTRACTS

September 9-13, 2019

INVITED TALKS

MODELS FOR THE OIL INDUSTRY

Yves Achdou
Université Paris-Diderot

This talk will be devoted to models for the oil industry. This is collaboration with C. Bertucci, J-M. Lasry, P-L. Lions, A. Rostand and J. Scheinkman.
The first model that will be discussed is a long term model with a monopolist (OPEC) facing a competitive fringe of minor producers. This is a first departure from the Hotelling framework, who considered only a monopolist. We make a more radical departure from the Hotelling framework by considering that new reserves are the result of research efforts that are endogenously determined, as we did in a previous work on mining resources [1]. The minor producers investments are bound by borrowing constraints. We end up with a model coupling a major agent and a crowd of minor agents playing a mean field game. The latter model is described by a system of PDEs coupling a HJB equation and a finite dimensional master equation. We briefly discuss numerical simulations, calibration, and the outputs of the model.
The second model that will be presented is a short term model in which the monopolist faces a crowd of arbitragists who own the speculative storage facilities. The model is described by a system of two PDEs, and a new kind of state constraint boundary condition for the system of PDEs plays a key role. The numerical simulations show the existence of cycles.

References

[1] Y. Achdou, P.N. Giraud, J-M. Lasry, P-L. Lions, A Long-Term Mathematical Model for Mining Industries *Applied Mathematics and Optimization* 74 (2016), no. 3, 579-618.

[2] Lasry, J. M., & Lions, P. L. Mean-field games with a major player. *Comptes Rendus Mathematique* 356(8), (2018), 886-890.

[3] Cardaliaguet, P., Cirant, M., & Porretta, A. Remarks on Nash equilibria in mean field game models with a major player. *arXiv preprint arXiv:1811.02811.* (2018).

LARGE TOURNAMENT GAMES

Erhan Bayraktar
University of Michigan

We consider a stochastic tournament game in which each player is rewarded based on her rank in terms of the completion time of her own task and is subject to cost of effort. When players are homogeneous and the rewards are purely rank dependent, the equilibrium has a surprisingly explicit characterization, which allows us to conduct comparative statics and obtain explicit solution to several optimal reward design problems. In the general case when the players are heterogenous and payoffs are not purely rank dependent, we prove the existence, uniqueness and stability of the Nash equilibrium of the associated mean field game, and the existence of an approximate Nash equilibrium of the finite-player game. Our results have some potential

economic implications; e.g., they lend support to government subsidies for R and D, assuming the profits to be made are substantial. Joint work with Jaka Cvitanić and Yuchong Zhang

---

MEAN FIELD APPROACH TO STOCHASTIC CONTROL WITH PARTIAL INFORMATION

Alain Bensoussan
University of Texas at Dallas

The classical stochastic control problem under partial information can be formulated as a control problem for Zakai equation, whose solution is the unnormalized conditional probability distribution of the state of the system, which is not directly accessible. Zakai equation is a stochastic Fokker-Planck equation. Therefore, the mathematical problem to be solved is very similar to that met in Mean Field Control theory. Since Mean Field Control theory is much posterior to the development of Stochastic Control with partial information, the tools, techniques and concepts obtained in the last decade, for Mean Field Games and Mean field type Control theory, have not been used for the control of Zakai equation. It is the objective of this work to connect the two theories. Not only, we get the power of new tools, but also we get new insights for the problem of stochastic control with partial information. For mean field theory, we get new interesting applications, but also new problems. The possibility of using direct methods is, of course, quite fruitful. Indeed, if Mean Field Control Theory is a a very comprehensive and powerful framework, it leads to very complex equations, like the Master equation, which is a nonlinear infinite dimensional P.D.E., for which general theorems are hardly available, although an active research in this direction is performed. Direct methods are particularly useful to obtain regularity results. We will develop in detail the linear quadratic regulator problem, but because we cannot just consider the gaussian case , well know results , like the separation principle are not available. An interesting and important result is available in the literature, due to A. Makowsky. It describes the solution of Zakai equation for linear systems with general initial condition (non-gaussian). Curiouly , this result had not been exploited for the control aspect, in the literature. We show that the separation principle can be extended for quadratic pay-off functionals, but the Kalman filter is much more complex than in the gaussian case.

MEAN FIELD GAMES WITH OPTIMAL STOPPING, IMPULSE CONTROL, AND MORE

Charles Bertucci
Université Paris Dauphine

In this talk, we first consider mean field games in which the players can decide to exit the game. We build a suitable notion of solutions for such games : Nash equilibria in mixed strategies modeled by some sorts of variational inequalities. We then explain why this previous notion can be generalized to address different (but related) questions such as mean field games with impulse controls, numerical methods, the presence of a major player and so on.

LARGE POPULATION GAMES IN UNBOUNDED NETWORKS AND THE GRAPHON MEAN FIELD GAME EQUATIONS

Peter Caines
McGill University

Very large networks linking dynamical agents are now ubiquitous and there is significant interest in their analysis, design and control. The emergence of the graphon theory of large networks and their infinite limits has recently enabled the formulation of a Graphon Control (GC) theory of the centralized control of dynamical systems distributed on asymptotically infinite networks [Gao and Caines, IEEE CDC 2017, 2018]. Furthermore, the study of non-cooperative dynamical games on unbounded networks has recently been initiated in [Caines and Huang, IEEE CDC 2018] where the Graphon Mean Field Games (GMFG) are formulated. In this talk the GMFG framework will first be presented, followed by the basic existence and uniqueness results for the GMFG equations and an epsilon-Nash theorem relating infinite population equilibria on infinite networks to certain approximating finite population equilibria on finite networks. Finally, the special case of Linear Quadratic Gaussian (LQG) GMFG systems will be sketched for some simple graphons.

Peter E. Caines, McGill University, Montreal
Minyi Huang, Carleton University, Ottawa

A MEAN FIELD GAMES APPROACH TO CLUSTER ANALYSIS

Fabio Camilli
SBAI, Università di Roma, Sapienza

A finite mixture model, given by a convex combination of probability density functions, is a powerful probabilistic tool for statistical modeling of data, with applications to pattern recognition, computer vision, signal and image analysis, machine learning, etc. Given a finite data set, the corresponding finite mixture model can be computed by means of the Expectation-Maximization (EM) algorithm, a classical optimization technique which incrementally converges to a maximum likelihood estimate of the model's parameters. In this talk, I will present a multi-population Mean Field Games systems which can be intepreted as an infinite-dimensional version of the EM algorithm. I will discuss the theoretical aspects of the method and the application to some problems in cluster analysis.

L.Aquilanti, S.Cacace, F.Camilli, R.De Maio, A Mean Field Games approach to cluster analysis, arXiv:1907.02261

FIRST ORDER MEAN FIELD GAMES WITH STATE CONSTRAINTS

Piermarco Cannarsa
Università di Roma Tor Vergata

This talk will mainly focus on deterministic mean field games in the presence of state constraints. Due to the appearance of singular parts in the distribution of players, the typical methods used to prove the existence and uniqueness of solutions are no longer available. This obstruction motivates the introduction of relaxed equilibria, which are defined adopting a Lagrangian formulation of the problem. This idea was used in three recent papers in collaboration with R.

Capuani and P. Cardaliaguet, where the related pdf system with the natural boundary conditions for constrained problems was also recovered. A further step in the analysis is the study of the asymptotic behaviour of constrained solutions as time goes to infinity, which is the subject of a joint work with W. Cheng, C. Mendico, and K. Wang.

WEAK KAM THEORY FOR POTENTIAL MEAN FIELD GAMES

Pierre Cardaliaguet
Université Paris Dauphine

In this joint work with Marco Masoero, we develop the counterpart of weak KAM theory for potential mean field games. This allows to describe the long time behavior of time-dependent potential mean field game systems. Our main result is the existence of a limit, as time tends to infinity, of the value function of an optimal control problem stated in the space of measures. In addition, we show a mean field limit for the ergodic constant associated with the corresponding Hamilton-Jacobi equation.

STOCHASTIC GRAPHON GAMES: THE STATIC CASE

René Carmona
Princeton University

We introduce a class of static games with a continuum of players as limits of finite player static games for which player sget idiosyncratic random signals. We analyze the limits as graphon games and we emphasize the differences with static mean field games. Joint work with D. Cooney, C. Graves and M. Lauriere.

HETEROCLINIC AND PERIODIC CONNECTIONS FOR A FIRST ORDER MEAN FIELD GAME WITH DENSITY CONSTRAINT AND AGGREGATING POTENTIAL

Annalisa Cesaroni
Università di Padova

I will discuss some results about of a first order variational mean field game with a constraint on the maximal density of the distribution of players, and an aggregating potential given by a confining function and by a nonlocal interaction potential of Riesz type. I analyze the system using the variational interpretation of the system as optimality conditions of a energy functional on the space of continuous curves with values in an appropriate Wasserstein space. Under the assumption that the system is symmetric with respect to a reflection and that there are two only minimizers of the stationary system, I discuss existence and some properties of minimal heteroclinic connections, that is minimizers of the energy functional connecting the 2 ground states of the stationary system, and of periodic connections, that is periodic minimizers of the energy functional. Moreover, the periodic minimizers are shown to converge, in a suitable sense, to heteroclinic connections, as the period goes to infinity. The results I will present are contained in a work in progress with M. Cirant (Parma).

RESTORATION OF UNIQUENESS FOR MEAN FIELD GAMES ON A FINITE STATE SPACE WITH A COMMON NOISE

François Delarue
Université Nice Sophia Antipolis

We here discuss how a common noise may help for restoring uniqueness to mean field games on a finite state space. The key point is to address the unique solvability of the corresponding master equation on the finite dimensional simplex and to invoke earlier results on the smoothing properties of diffusions with values in manifolds with corners. Interestingly enough, our result opens the door to selection principles for standard mean field games on a finite state space without uniqueness. This question is addressed in a companion talk by A. Cecchin in the framework of potential games.

This is a joint work with E. Bayraktar, A. Cecchin and A. Cohen

---

CORRELATED EQUILIBRIA AND MEAN FIELD GAMES

Markus Fischer
Università di Padova

Mean field games are limit models for symmetric N-player games, as N tends infinity, where the prelimit models are solved in terms of Nash equilibria. A generalization of the notion of Nash equilibrium, due to Robert Aumann (1973, 1987), is that of a correlated equilibrium. Here, we discuss, in a simple non-static setting, the mean field game limit for correlated equilibria. We give a definition of correlated mean field game solution, prove that it arises as limit of N-player correlated equilibria in restricted ("open-loop") Markov feedback strategies, and show how to construct approximate N-player equilibria starting from a correlated mean field game solution. We also compare our definition to the one by Lacker (2018) of weak solutions for mean field games without a common noise. (Joint work with Luciano Campi, The London School of Economics and Political Science.)

SELECTION PRINCIPLES FOR THE VANISHING DISCOUNT MFG PROBLEM

Diogo Gomes
KAUST

Here, we study the existence and the converge of solutions for the vanishing-discount MFG problem with a quadratic Hamiltonian. We give conditions under which the discounted problem has a unique classical solution and prove convergence of the vanishing-discount limit to a unique solution. Then, we establish refined asymptotics for the limit. When those conditions do not hold, the limit problem may not have a unique solution and its solutions may not be smooth, as we illustrate in an elementary example. Finally, we investigate the stability of regular weak solutions and address the selection problem. Using ideas from Aubry-Mather theory, we establish a selection criterion for the limit.

MINIMAL-TIME MEAN-FIELD GAMES

Guilherme Mazanti
Université Paris Sud

Motivated by the problem of proposing mean field game models for crowd motion, this talk considers a mean field game where agents want to leave a bounded domain through a part of its boundary in minimal time. Each agent is free to move in any direction, but their maximal speed is assumed to be bounded in terms of the distribution of agents around their position in order to model congestion phenomena. With respect to most mean field game models in the literature, the novelties here are the velocity constraint and the fact that the final time in the optimization criterion is not fixed, which are important features from a modeling point of view for crowd motion but bring several extra difficulties in the analysis.

After presenting the model and its motivation, we establish the existence of equilibria for this game using a fixed-point strategy based on a Lagrangian formulation. In the case where agents may leave the domain through any point of its boundary, thanks to some further regularity properties of optimal trajectories obtained through Pontryagin's maximum principle, we characterize equilibria through a MFG system. We also present some simulations in simple situations and discuss other recent results and ongoing work, including sufficient conditions ensuring the L^p regularity of the distribution of agents and the case where agents' dynamics are stochastic.

This talk is based on joint works with Samer Dweik and Filippo Santambrogio.

MEAN FIELD GAMES MODELING CROWD CONGESTION

Alessio Porretta
Università di Roma Tor Vergata

In his courses at College de France, P.-L. Lions introduced sample models of mean field games with congestion, by penalizing the cost (of motion) of the agents in a proportional way as the density of the crowd increases. Those models have peculiar interest in terms of PDEs, because the usual MFG system does not have a variational structure in this case. In a joint work with Yves Achdou, we proved (AIHP, 2018) existence and uniqueness of weak solutions for second order MFG systems of this kind. In this talk I will discuss new results that we have and further work in progress concerning the vanishing viscosity limit and the first order system.

A VARIATIONAL APPROACH TO THE MEAN FIELD PLANNING PROBLEM

Giuseppe Savaré
Università di Pavia

We precent some results concerning a first-order mean field planning problem associated to a convex Hamiltonian with quadratic growth and a monotone interaction term with polynomial growth. We exploit the variational structure of the system, which encodes the first order optimality condition of a convex dynamic optimal entropy-transport problem with respect to the unknown density and of its dual, involving the maximization of an integral functional among all the subsolutions of an Hamilton-Jacobi equation. Combining ideas from optimal transport, convex analysis and renormalized solutions to the continuity equation, we will prove existence and (at least partial) uniqueness of a weak solution. A crucial step of our approach relies on a careful

analysis of distributional subsolutions to Hamilton-Jacobi equations and on a measure-theoretic description of the optimality via a suitable contact-defect measure. (In collaboration with Carlo Orrieri and Alessio Porretta)

HOMOGENIZATION OF MEAN FIELD GAMES–STEP 1

Panagiotis Souganidis
University of Chicago

A VARIATIONAL APPROACH FOR THE PLANNING PROBLEM IN MEAN FIELD GAMES

Daniela Tonon
Université Paris Dauphine

The existence and uniqueness of weak solutions for standard first order and second order degenerate Mean Field Games systems have been proved through variational techniques by Cardaliaguet and collaborators. The idea consists in characterizing weak solutions as minimizers of two optimal control problems in duality. This strategy was introduced for the Monge-Kantorovich mass transfer problem by Benamou and Brenier and then extended by Carlier, Cardaliaguet and Nazaret. In this talk we discuss how these techniques can be applied to solve the planning problem in MFG (i.e. when the initial and final densities of the population are prescribed) and the penalized terminal payoff MFG system. We also show Sobolev estimates on the solutions.

PATTERN FORMATION IN COMPETITION-DIFFUSION SYSTEMS

Gianmaria Verzini
Politecnico di Milano

Several physical phenomena can be described by a certain number of densities (of mass, population, probability, ...) distributed in a domain and subject to laws of diffusion, reaction, and competitive interaction. Whenever the competitive interaction is the prevailing phenomenon, the densities can not coexist in the whole spatial region, and they tend to segregate; in the singular perturbation limit, this determines a partition of the domain. I will discuss some older and more recent results about regularity and qualitative properties of this kind of partitions. I will also try to enlighten some relations between this topic and the theory of MFG.

CONTRIBUTED TALKS

SELECTION BY VANISHING COMMON NOISE FOR POTENTIAL FINITE STATE MEAN FIELD GAMES

Alekos Cecchin
Université Nice Sophia Antipolis

By adding a suitable Wright-Fischer common noise in the dynamics of the law, uniqueness is restored for finite state mean field games, for any shape of the costs. Here we deal with a class of mean field games in which the costs admit a potential in the simplex of probability measures. We first address the relation between the unique solution of the randomized mean field game and the necessary condition for optimality of the unique minimizer of a suitable stochastic control problem stated in the simplex, called potential randomized game. Then we study the limit of these optimizers, as the intensity of the common noise vanishes, and show convergence to the mean field game solutions which are minimizers of the corresponding potential game (not randomized), that is a deterministic optimal control problem in the simplex. Joint work with François Delarue.

THE SOCIAL PLANNER PROBLEM WITH FINITE STATE SPACE

Asaf Cohen
University of Michigan

We consider a fi nite state continuous-time social planner problem with symmetric N agents that are weakly interacting. At any time instant, the social planner controls the rate of transition of the agents, aiming to minimize a cost. In addition to the control, agents move from one state to another according to an external force; both the transition rates and the cost depend on the current time and the empirical distribution of the agents over the states. We set up a (limiting) diffusion control problem that governs the limiting behavior of the N-agent problem in the following sense: the set of optimal controls in the limiting problem is exactly the set of all the limit points of all the asymptotic optimal control sequences in the N-agent problem. Moreover, we show a possible connection between the limiting problem and a nonlinear Kimura partial differential equation. Joint with Erhan Bayraktar.

SUBMODULAR MEAN FIELD GAMES: EXISTENCE AND APPROXIMATION OF SOLUTIONS

Jodi Dianetti
Universität Bielefeld

We study mean field games with scalar Ito-type dynamics and costs that are submodular with respect to a suitable order relation on the state and measure space. The submodularity assumption has a number of interesting consequences. Firstly, it allows us to prove existence of solutions via an application of Tarski's fixed point theorem, covering cases with discontinuous dependence on the measure variable. Secondly, it ensures that the set of solutions enjoys a lattice structure: in particular, there exist a minimal and a maximal solution. Thirdly, it guarantees the convergence to the mean field game for a simple learning procedure based on the best response map. The mean field game is first defined over ordinary stochastic controls, then extended to relaxed controls.

ON CLASSICAL SOLUTIONS OF TIME-DEPENDENT FRACTIONAL MEAN FIELD GAME SYSTEMS

Olav Ersland
NTNU

We consider parabolic Mean Field Game systems with nonlocal/fractional diffusion. Such models come from games where the noise is non-Gaussian and the resulting controlled diffusion process anomalous. Here the noise is modeled by Levy processes that are (close to) σ-stable processes. The corresponding diffusion operators/generators are then (close to) fractional Laplacians $(-\Delta)^{\frac{\sigma}{2}}$. Our main result is existence and uniqueness for classical solutions of Mean Field Game systems for $\sigma \in (1, 2)$. This corresponds to the subcritical or strong diffusion case where the equations are nondegenerate parabolic. We consider both local and nonlocal couplings. The existence proof is an extension of the fixed-point argument introduced by P.-L. Lions. The new ingredients are fractional heat kernel estimates, regularity results for fractional Bellman and Fokker-Planck equations and results on (very) weak solutions of fractional Fokker-Planck equations.

This is a joint work with Espen R. Jakobsen.

REGULARITY RESULTS FOR VISCOUS HAMILTON-JACOBI EQUATIONS WITH L^p RIGHT-HAND SIDE

Alessandro Goffi
Gran Sasso Science Institute/Università di Padova

In this talk, I will discuss recent developments on regularity properties of solutions to time-dependent Hamilton-Jacobi equations obtained in collaboration with M. Cirant. In particular, I will focus on Lipschitz regularity of solutions to viscous Hamilton-Jacobi equations with superlinear growth in the gradient and unbounded right-hand side. This analysis is partially motivated by problems arising in Mean-Field Games, where solutions of the so-called mean field equations are related to an optimal control problem for a Hamilton-Jacobi-type PDE having a right-hand side controlled only in some Lebesgue norm. The result is achieved via the so-called nonlinear adjoint method and is based on the regularity of the gradient of solutions to a (dual) Fokker-Planck equation.

MEAN FIELD GAMES OF CONTROLS: THEORY AND NUMERICAL SIMULATIONS

Z. Kobeissi
Laboratoire Jacques-Louis Lions, Univ. Paris Diderot

We consider a class of mean field games in which the optimal strategy of a representative agent depends on the statistical distribution of the states and controls. The existence and uniqueness results presented in this talk can be found in [1]. The numerical simulations shown are part of a joint work in progress with Y. Achdou.

References

[1] Z. Kobeissi *On Classical Solutions to the Mean Field Game System of Controls*, arXiv e-prints arXiv:1904.11292 (2019).

MACHINE LEARNING METHODS FOR MEAN FIELD CONTROL AND GAMES

Mathieu Lauriere
Princeton University

In this talk, we propose several methods for mean field control problems and mean field games, both in the ergodic setting and the finite time horizon setting. These methods are based on machine learning tools such as function approximation via neural networks and optimization relying on stochastic gradient descent. We investigate the numerical analysis of these methods and prove bounds on the approximation error and the generalization error. We then consider numerical test cases, including examples in large dimension or with common noise. If time permits, we will also present generalizations of these methods to solve numerically the Master equation. This is joint work with Rene Carmona (Princeton University).

A MEAN FIELD APPROACH TO MODEL FLOWS OF AGENTS WITH PATH PREFERENCES OVER A NETWORK

Rosario Maggistro
Università Ca' Foscari, Venezia

In this talk we address the problem of modeling the traffic flow of a heritage city whose streets are represented by a network. We consider a mean field approach where the standard forward backward system of equations is also intertwined with a path preferences dynamics. The path preferences are influenced by the congestion status on the whole network as well as the possible hassle of being forced to run during the tour. We prove the existence of a mean field equilibrium as a fixed point, over a suitable set of time-varying distributions, of a map obtained as a limit of a sequence of approximating functions. Then, a bi-level optimization problem is formulated for an external controller who aims to induce a specific mean field equilibrium. This is a joint work with Fabio Bagagiolo (Università di Trento) and Raffaele Pesenti (Università Ca' Foscari, Venezia).

WEAK KAM THEORY FOR POTENTIAL MFG

Marco Masoero
Université Paris Dauphine

We develop the counterpart of weak KAM theory for potential mean field games. This allows to describe the long time behavior of time-dependent potential mean field game systems. Our main result is the existence of a limit, as time tends to infinity, of the value function of an optimal control problem stated in the space of measures. In addition, we show a mean field limit for the ergodic constant associated with the corresponding Hamilton-Jacobi equation.

MEAN FIELD GAMES EQUILIBRIA FOR LINEAR CONTROL SYSTEMS

Cristian Mendico
GSSI L'Aquila and Paris Dauphine University

In this talk I am going to present recent results about Mean Field Games for linear control systems. More precisely, the dynamics of each agents is given by a linear controlled ODE on $\mathbb{R}^d$ for controls functions in $L^2(0,T;\mathbb{R}^k)$. Using the Lagrangian formulation of the problem, we are able to define and prove the existence, and uniqueness, of the so called mean field games equilibria. Moreover, I present some regularity results on the mean field games equilibria and on the value function of the associated optimal control problem. That is, any general equilibrium generates a Hölder continuous family of probability measures and consequently, the value function is locally Lipschitz continuous and locally semiconcave with fractional modulus. In conclusion, under some extra assumptions we are able to prove that there exists at least one mean field games equilibrium which generates a Lipschitz continuous family of probability measures.

References

[1] P. Cannarsa, C. Mendico, Existence, uniqueness and regularity of mean field games equilibria for linear control systems, in-preparation.

Joint work with: Piermarco Cannarsa, (*University of Rome "Tor Vergata"*).

LAGRANGIAN, EULERIAN AND KANTOROVICH FORMULATIONS OF MULTI-AGENTS SYSTEMS

Carlo Orrieri
Università di Trento

The aim of the talk is to study the relations among various formulations of an optimal control problem for interacting particles systems. Starting with a random evolution of N particles we firstly show the equivalence of the Lagrangian and Eulerian formulations at the level of the value functions. Then we introduce a general setting and we discuss the Γ-convergence as the number of particles diverges $N \uparrow +\infty$. To deal with optimal control problems in space of measure we take advantage of the optimal transportation theory and of the superposition principle. This is a joint work with Giulia Cavagnari, Stefano Lisini and Giuseppe Savaré.

MEAN-FIELD MODELS WITH MULTISCALE STRUCTURE

Guglielmo Pelino
Università di Padova

A natural way of going beyond mean-field models consists in considering a population comprised by many communities, each containing many individuals. The interactions among individuals, of mean-field type within a single community, suitably scale when individuals belong to different communities. This gives rise to space-time multiscaling phenomena that are well understood in the case of interacting Wright-Fisher diffusions, leading to rigorous renormalization group arguments. We formulate an example concerning Ising-type models, giving partial results and open problems.

MEAN FIELD GAMES UNDER INVARIANCE CONDITIONS FORTHE STATE SPACE

Michele Ricciardi
Università di Roma Tor Vergata

In this talk, based on a work with Alessio Porretta, we study a particular class of Second-Order Mean Field Games, where the dynamic of each player is confined into a compact domain. These leads to a system where the uniform ellipticity of the diffusion is no longer guaranteed. We give here a suitable definition of weak solutions and we prove existence and uniqueness results. Strengthening the hypotheses on the data, we obtain further regularity for the value function and for the density of the players. Finally, we extend these results for non-smooth domains.

MEAN-FIELD LANGEVIN DYNAMICS AND ENERGY LANDSCAPE OF NEURAL NETWORKS

David Siska
University of Edinburgh

We present a probabilistic analysis of the long-time behaviour of the nonlocal, diffusive equations with a gradient flow structure in 2-Wasserstein metric, namely, the Mean-Field Langevin Dynamics (MFLD). Our work is motivated by a desire to provide a theoretical underpinning for the convergence of stochastic gradient type algorithms widely used for non-convex learning tasks such as training of deep neural networks. The key insight is that the certain class of the finite dimensional non-convex problems becomes convex when lifted to infinite dimensional space of measures. We leverage this observation and show that the corresponding energy functional defined on the space of probability measures has a unique minimiser which can be characterised by a first order condition using the notion of linear functional derivative. Next, we show that the flow of marginal laws induced by the MFLD converges to the stationary distribution which is exactly the minimiser of the energy functional. We show that this convergence is exponential under conditions that are satisfied for highly regularised learning tasks. At the heart of our analysis is a pathwise perspective on Otto calculus used in gradient flow literature which is of independent interest. Our proof of convergence to stationary probability measure is novel and it relies on a generalisation of LaSalle's invariance principle. Importantly we do not assume that interaction potential of MFLD is of convolution type nor that has any particular symmetric structure. This is critical for applications. Finally, we show that the error between finite dimensional optimisation problem and its infinite dimensional limit is of order one over the number of parameters.

Joint work with Kaitong Hu, Zhenjie Ren and Lukasz Szpruch.

EXISTENCE OF WEAK SOLUTIONS TO TIME-DEPENDENT MEAN-FIELD GAMES

Teruo Tada
KAUST

We establish the existence of weak solutions to time-dependent monotone mean-field games (MFGs). Our method applies to a general setting of problems; first- or second-order, local or non-local coupling. First, we introduce a monotone high-order regularized elliptic problem though the original problem is a parabolic type. Then, we investigate properties of weak solutions to the regularized MFG. Second, by Schaefer's fixed-point theorem and monotonicity, we prove that there exists a unique weak solution to the regularized problem. Finally, we take the limit

of the solutions to the regularized problem, we verify the existence of a weak solution to the original MFG.

TIME-FRACTIONAL MEAN FIELD GAMES

Qing Tang
China University of Geosciences, Wuhan

The theory of time-fractional mean field games was introduced in 2017-2018 by Camilli and De Maio to study MFG systems with memory effects and subdiffusion structure (Continuous Time Random Walks, CTRW). Subdiffusion is a type of non-Markovian anomalous diffusion process which can be caused by trapping effects in particle (agent) dynamics and is widely observed in molecular biology, finance etc. I will focus on a variational formulation of fractional MFGs, i.e. deriving a fractional MFG system via a Benamou-Brenier type optimal transport problem modified with memory effect. I will also discuss the potential applications in economics and finance. This is based on preprint of my joint work with Fabio Camilli and work in progress of myself.

Q. Tang and F. Camilli. Variational time-fractional mean field games. arXiv 1812.05431, 12 2018.

TWO-SCALE HOMOGENIZATION OF A STATIONARY MEAN-FIELD GAME

Xianjin Yang
KAUST

In this paper, we characterize the asymptotic behavior of a first-order stationary mean-field game (MFG) with a logarithm coupling, a quadratic Hamiltonian, and a periodically oscillating potential. This study falls into the realm of the homogenization theory, and our main tool is the two-scale convergence. Using this convergence, we rigourously derive the two-scale homogenized and the homogenised MFG problems, which encode the so-called macroscopic or effective behavior of the original oscillating MFG. Moreover, we prove existence and uniqueness of the solution to these limit problems.

DISASTER RISK IN INCOMPLETE MARKETS: LARGE DEVIATIONS FROM APPROXIMATE AGGREGATION

Jingjie Zhang
University of Michigan

A folk-wisdom in the incomplete markets literature is that market incompleteness makes little quantitative difference in the dynamics of aggregate quantities and prices. Agents facing the inability to hedge idiosyncratic risk build buffer savings which mute the effect of idiosyncratic shocks. The behavior of aggregate quantities and prices in such an economy is quantitatively similar to an economy with complete markets where the cross-section of agents can be aggregated to a single representative agent. We show that such approximate aggregation breaks down when we allow for disaster risk: the potential for low probability, large negative shocks. We illustrate

this in the context of the standard Aiyagari model, extended to include an aggregate shock to total factor productivity. Markets are incomplete because agents have a single asset (capital) to hedge two sources of risk: the shock to aggregate productivity which changes its value at a known time and un-insurable labor income risk which follows a mean-reverting diffusive process. We use methods from mean-field games to numerically solve for the exact equilibrium for a range of possible values of the size of the disaster shock.

To quantify the effect of disaster risk, we compare aggregate savings in four related economies with the same production technologies and agent preferences. These economies differ along two dimensions: the extent to which idiosyncratic risk can be shared across agents and the amount of aggregate disaster risk faced by agents. In the economy with (without) disaster risk, total factor productivity take three (two) possible values. To make the aggregate shock comparable between the disaster and non-disaster economies, we choose parameters so that the first two moments of the aggregate shocks are the same. The amount of disaster risk is then determined by the value of the shock in the disaster state. To quantify the effect of market incompleteness on aggregate quantities in the presence of disaster risk, we compute the ratio of the increase in aggregate capital stock due to the presence of disaster risk when markets are incomplete to the same increase in capital stock when markets are complete. We find that this ratio increases non-linearly with the size of the disaster shock. The magnitude of the sensitivity of this ratio depends sensitively on the size of disasters.