

Monge-Ampere equations and applications

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The Monge-Ampère equation (MA) has the form $\det D^2u = f$. In this series of lectures we present an introduction this equation and some applications. In general Monge-Ampère type equations involve the Jacobian determinant of a map, and arise in the mathematical description of numerous geometric, optical, acoustic, and electromagnetic applications, in particular, in lens and reflector antenna design and in optimal mass transportation. I will try to emphasize ideas and techniques that might be applicable to other contexts.

A tentative program is the following:

1. Construction of generalized solutions to the MA equation and its basic properties.
2. Solution of the Dirichlet problem for MA.
3. The theory of sections of solutions to MA and their role in the regularity theory: $W^{2,p}$ -estimates and Hölder estimates of second derivatives; covering arguments with the sections.
4. Applications and extensions of some of these ideas to solve problems in geometric optics.

Prerequisites: Functions of real variable, Lebesgue measure and integration. Basic knowledge of pdes at the level of parts I and II of Evans' book.

References

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