

# Extremum and isoperimetric problems for eigenvalues of the Laplacian

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In this series of lectures, we will discuss the following question: let  $a \leq b$  be two given positive numbers. Is there exists a plane open set  $\Omega$  of given area such that  $a$  and  $b$  are the first two eigenvalues of the Laplace operator with Dirichlet boundary condition on  $\Omega$ ? Is there exists a convex one? What happens if we replace the Dirichlet boundary condition by the Neumann one? This simple questions will lead us to (re)visit classical isoperimetric inequalities for the first eigenvalues of the Laplacian which is the simplest differential operator and the links between the eigenvalues and the geometry of the domain. It will also be the occasion to introduce and use classical tools of modern analysis like Schwarz and Steiner rearrangement, variational methods,  $\gamma$ -convergence, derivative with respect to the domain.