Wellposedness of Linear Control Systems

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May 16, 2013

We consider linear control systems with unbounded control and output operators and investigate well-posedness with a semigroup approach.

On the Banach spaces X, U, Y, called *state, control* and *output space*, we consider the following linear operators.

- $A : D(A) \subset X \to X$, generator of a C_0-semigroup, the *state operator*,
- $B \in \mathcal{L}(U, X_{-1})$[1] the *control operator*,
- $C \in \mathcal{L}(X_1, Y)$[2] the *output operator*,
- $D \in \mathcal{L}(U, Y)$ the *feedthrought operator*.

With these operators we introduce the following linear control system

$$\Sigma(A, B, C, D) : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t), & t \geq 0, \\ y(t) = Cx(t) + Du(t), & t \geq 0. \end{cases}$$

We construct an operator matrix $\mathcal{A}$ on an appropriate product space $\mathfrak{X}$ and define the system $\Sigma(A, B, C, D)$ to be well-posed if $\mathcal{A}$ generates a strongly continuous semigroup on $\mathfrak{X}$.

The generator property of $\mathcal{A}$ can be characterized by means of certain properties "admissibility" of the operators B, C.

[1] X_{-1} is the extrapolation space of X w.r.t. A.

[2] X_1 denote $D(A)$ endowed with the graph norm induced by A.