SINGULAR NUMBERS OF CORRECT RESTRICTIONS OF NON-SELFADJOINT ELLIPTIC OPERATORS

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Joint work with M. Oterlbaev

Let $\mathcal{L}$ be an elliptic differential expression of the following form: for $u \in C^\infty(\mathbb{R}^n)$

$$(\mathcal{L}u)(x) = \sum_{|\alpha|,|\beta| \le l} (-1)^{|\alpha|+|\beta|} D^\alpha \left(A_{\alpha\beta}(x) D^\beta u\right), \quad x \in \mathbb{R}^n,$$

where $A_{\alpha\beta} \in C^l(\mathbb{R}^n)$ are real-valued functions for all multi-indices α, β satisfying $|\alpha|, |\beta| \le l$. Moreover, let, for a domain $\Omega \subseteq \mathbb{R}^n$, $L_\Omega : D(L_\Omega) \to L_2(\Omega)$ be a linear operator closed in $L_2(\Omega)$ generated by $\mathcal{L}$ on Ω. A restriction $A : D(A) \to L_2(\Omega)$ of L_Ω is *correct* if the equation $Au = f$ has a unique solution $u \in D(A)$ for any $f \in L_2(\Omega)$ and the corresponding inverse operator $A^{-1} : L_2(\Omega) \to D(A)$ is bounded.

Let A and B be compact linear operators in a Hilbert space H. If there exist $0 < \alpha < \beta$ and $c_1, c_2 > 0$ such that for singular numbers $s_k(A)$ and $s_k(B)$ the conditions $c_1 k^{-\alpha} \le s_k(A)$, $s_k(B) \le c_2 k^{-\beta}$, hold, we say that in the representation $C = A + B$ the operator A is a *leading* operator and the operator B is a *non-leading* operator.

Theorem 1. *Let $l, n \in \mathbb{N}$, $n \ge 2$, $2l(1 - \frac{1}{n}) < s \le 2l$, and Ω be a bounded domain in $\mathbb{R}^n$ with the boundary $\partial\Omega$ of class C^{2l}. Moreover, let A and B be correct restrictions of the operator L_Ω such that $D(A) \subseteq W_2^{2l}(\Omega)$, $D(B) \subseteq W_2^s(\Omega)$ and the operators $A^{-1} : L_2(\Omega) \to W_2^{2l}(\Omega)$, $B^{-1} : L_2(\Omega) \to W_2^s(\Omega)$ are bounded. Then in the representation $B^{-1} = A^{-1} + K$ the operator A^{-1} is a leading operator and the operator $K = B^{-1} - A^{-1}$ is a non-leading operator.*

Theorem 2. *Let $l, n \in \mathbb{N}$, $n \ge 2$, $2l(1 - \frac{1}{n}) < s \le 2l$ and Ω be a bounded domain in $\mathbb{R}^n$ with the boundary $\partial\Omega$ of class C^{2l}. Then there exists $b > 0$ such that the singular numbers $s_k(B)$ of all correct restrictions B of the operator L_Ω satisfying the condition $D(B) \subseteq W_2^s(\Omega)$ with the bounded inverse $B^{-1} : L_2(\Omega) \to W_2^{2l}(\Omega)$*

$$\lim_{k\to\infty} s_k(B) k^{-\frac{2l}{n}} = b\,.$$

Also other related results, formulated in [1], will be presented in the talk.

[1] V.I. Burenkov, M. Otelbaev, *On the singular numbers of correct restriction of elliptic differential operators.* Eurasian Math. J. 2 (2011), no. 1, 145 – 148.