Andrea Cianchi

Title: Sharp Sobolev embeddings and rearrangements

Abstract. A classical result, known as Pólya-Szegö principle, asserts that radial symmetrization decreases norms of the gradient of Sobolev functions vanishing on the boundary of their domain, while preserving norms of functions. This property enables one to reduce various Sobolev type inequalities to considerably simpler one-dimensional problems, and has been exploited in the proof of well-known inequalities with sharp constants, such as those of Moser, Talenti and Aubin. In recent years, alternative methods have been introduced to derive reduction principles to one-dimensional inequalities for Sobolev type embeddings where symmetrization fails. This is the case, for instance, when functions which need not vanish on the boundary are concerned, or higher-order derivatives come into play, or non-Euclidean measures are considered, or trace inequalities are taken into account. The use of one-dimensional rearrangements is crucial in the proof of most of the reduction principles in question. After recalling the classical Pólya-Szegö principle, and a few of its applications, we shall discuss some of the developments mentioned above.