Composition Operators on Sobolev Spaces

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The composition operator $C_\varphi(f)$ is a linear operator defined by the composition rule $C_\varphi(f) := f \circ \varphi$. In the language of the category theory, the composition operator is a pull-back operator. The domain of definition of a composition operator is usually taken to be some Banach space. Classical examples are spaces of analytic functions, smooth functions, continuous functions. In all these cases homeomorphisms that induce composition operators have the same type of smoothness as function spaces.

For Sobolev spaces situation is completely different. Composition operators (pull back operators) for Sobolev spaces are special geometric classes of mappings (conformal, quasi-conformal mappings and its generalizations).

Let $\Omega \subset R^n$ be an Euclidian domain. The homogeneous Sobolev space $L^1_p(\Omega)$, $1 \leq p < \infty$, consists of locally summable, weakly differentiable functions $f : \Omega \to \mathbb{R}$ with the finite seminorm:

$$\|f \mid L^1_p(\Omega)\| = \Big(\int\limits_D |\nabla f|^p dx \Big)^{\frac{1}{p}}, \quad \nabla f = (\frac{\partial f}{\partial x_1}, ..., \frac{\partial f}{\partial x_n}).$$

Only conformaI homeomorphism induces isometries of spaces L^1_n. Isomorfisms of L^1_n is induced by quasi-conformal homeomorphisms. For $p \neq n$ an answer is more complicated. All these problems represent a subject of this work.

Two main applications of the composition operators on Sobolev spaces will be discussed also, i.e. embedding theorems for non-smooth domains and elliptic boundary value problems.