Boundedness and compactness of the integral operators in weighted Sobolev space

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Abstract.

Let $I=(a,b)$, $-\infty\le a<b\le\infty$. Let $1<p,q,r<\infty$, $\frac{1}{p}+\frac{1}{p'}=1$. Suppose that v , u ρ and w are non-negative and measurable functions on I such that v^p , u^r ρ^p , w^q , $\rho^{-p'}$ and $w^{-q'}$ are locally summable functions on I .

Let $L_{p,\rho}\equiv L_p(\rho,I)$ denote the space of measurable functions on I such that the norm $\|f\|_{p,\rho}\equiv\|\rho f\|_p$ is finite , where $\|\cdot\|_p$ - is the standard norm of the space $L_p(I)$.

We denote by $W^1_{p,r}(u,v)\equiv W^1_{p,r}(u,v,I)$ the set of locally absolutely continuous functions f on I with following finite norm

$$\|f\|_{W^1_{p,r}}=\|uf'\|_r+\|vf\|_p,$$

where $\|\cdot\|_p$ is the standard norm of the space $L_p\equiv L_p(I)$. In case $p=r$ and $u\equiv\rho$ we assume that $W^1_{p,p}(\rho,v)\equiv W^1_p(\rho,v)$, $\|f\|_{W^1_{p,p}}=\|f\|_{W^1_p}$.

Let $A\overset{\circ}{C}(I)$ be the set of locally absolutely continuous functions with compact supports on I . Denote by $\overset{\circ}{W}{}^1_p\equiv\overset{\circ}{W}{}^1_p(\rho,v)\equiv\overset{\circ}{W}{}^1_p(\rho,v,I)$ the closure of the set $A\overset{\circ}{C}(I)\bigcap W^1_p(\rho,v)$ with respect to the norm $\|f\|_{W^1_p}=\|\rho f'\|_p+\|vf\|_p$.

We consider the problem of boundedness and compactness from the weighted Sobolev $\overset{\circ}{W}{}^1_p(\rho,v)$ space into the weighted Sobolev $W^1_{p,r}(u,v)$ space of the integral operators

$$\mathcal{K}^+f(x)=\int\limits_a^x K(x,s)f(s)ds,\quad x\in I, \tag{1}$$

$$\mathcal{K}^-g(s)=\int\limits_s^b K(x,s)g(x)dx,\quad s\in I \tag{2}$$

with measurable kernel $K(\cdot,\cdot)\ge 0$ on $\Omega=\{(x,s):a<s\le x<b\}$.