

Harmonic analysis in the Ornstein-Uhlenbeck setting

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Abstract

The Ornstein-Uhlenbeck operator is given in \mathbb{R}^n by

$$Lf = -\Delta f + 2x \operatorname{grad} f.$$

It has an associated heat semigroup $(e^{-tL})_{t>0}$, defined by means of the spectral decomposition of L , which involves the classical Hermite polynomials. In many ways, this is analogous to the standard Laplacian and the Gaussian heat kernel. The Ornstein-Uhlenbeck setting is one of several important models of harmonic analysis. In these, one defines and studies analogs of notions from standard harmonic analysis, like maximal operators and singular integrals, in particular Riesz transforms. Instead of Lebesgue measure, one uses a measure adapted to the model; in the Ornstein-Uhlenbeck case the measure is Gaussian.

In this course, we shall introduce the Hermite polynomials and compute the kernel of the Ornstein-Uhlenbeck semigroup. To prove the pointwise continuity of the semigroup, i.e., $e^{-tL}f \rightarrow f$ as $t \rightarrow 0$ a.e., we shall study the corresponding maximal operator, aiming at the weak type (1,1) estimate.

If time allows, we will also describe some of the other models.