Minicourse on Steklov Eigenproblems and Boundary Traces.
University of Padua   June 2017

This minicourse will treat issues about the representation and properties of solutions of linear elliptic boundary value problems with **non-zero boundary conditions**. For simplicity the course will concentrate on the analysis of Laplacian boundary value problems. The prerequisite is material at the level of chapter 9 of Brezis' text on Functional Analysis, Sobolev spaces and Partial Differential Equations. (Springer).

Topics to be covered will include

I.   Inner Products and Orthogonal decompositions of real Hilbert-Sobolev spaces. Eigenvalue problems for bilinear forms on Hilbert spaces.

II   Steklov eigenproblems and the construction of orthogonal bases for null spaces of linear elliptic operators.

III.   The Spectral characterization of trace spaces, the properties and representations of boundary operators.

IV.   The representation and properties of solutions of homogeneous linear elliptic equations with non-zero boundary data. Inequalities, approximation of solutions and the dependence of solutions on boundary conditions.

V.   Reproducing Kernel Hilbert spaces of harmonic functions. The Reproducing kernel and the Poisson operator. The Singular Value Decomposition of the Poisson operator as a map of $L^2(\partial\Omega)$ to $L^2(\Omega)$.