An Introduction to the Mathematical Theory of Inverse Problems

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Abstract: The course consists of two parts. In the first part we will give an introduction into the basis features and challenges of the area of inverse problems. We will give a couple of typical examples and explain why inverse problems are most often improperly posed. We will formulate these problems in the framework of functional analysis as the problem of solving (linear) equations $Kx = y^\delta$ with compact operators K and where y^δ are perturbations of the exact right hand side y.
In the second part we will consider Tikhonov's regularization method as the most important example of a regularization strategy. Tikhonov's method – as regularization methods in general – replaces the operator equation $Kx = y^\delta$ by an equation with a boundedly invertible operator involving a (small) parameter α. We will see how to choose α in dependence on the error δ of the right hand side and will study the problem of convergence when δ tends to zero.