# Uniqueness and non–uniqueness of prescribed mass NLS ground states on metric graphs

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**Abstract**

This talk addresses the problem of uniqueness of ground states of prescribed mass for the Nonlinear Schrödinger Energy with power nonlinearity on noncompact metric graphs with half–lines. We first show that, up to an at most countable set of masses, all ground states at given mass share the same frequency, that is the Lagrange multiplier appearing in the NLS equation is constant on the set of ground states of mass $\mu$. On the one hand, we apply this result to prove uniqueness of ground states on two specific families of noncompact graphs. On the other hand, we construct a graph that admits at least two ground states with the same mass having different Lagrange multipliers. This shows that the result for Lagrange multipliers is sharp in general, in the sense that one cannot get rid of the at most countable set of masses where it may fail without further assumptions. Our proofs are based on careful variational arguments and rearrangement techniques, and hold both for the subcritical regime $p \in (2, 6)$ and in the critical case $p = 6$. The content of the talk is based on the paper Dovetta S., Serra E., Tilli P., *Advances in Mathematics* (2020).