Long-time asymptotics for nonlocal porous medium equation with absorption or convection

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A large variety of models for conserved quantities in continuum mechanics or physics are described by the continuity equation $u_\tau + \nabla \cdot (u\mathbf{v}) = 0$, where the density distribution $u(y,\tau)$ evolves in time τ following a velocity field $\mathbf{v}(y,\tau)$. According to Darcy's law, the velocity $\mathbf{v}$ is usually derived from a potential p in the form $\mathbf{v} = -\mathcal{D}\nabla p$ for some tensor $\mathcal{D}$. In porous media, the power-law relation $p = u^m$ is commonly proposed. Although local constitutive relations like $p = u^m$ were successful in numerous practical models, there are situations where the potential (or pressure) p depends non-locally on the density distribution u. The simplest prototypical example is $p = (-\Delta)^{-s}u$, expressed as the Riesz potential of u. The resulting evolution equation then becomes

$$u_\tau - \nabla \cdot (u\nabla(-\Delta)^{-s}u) = 0, \tag{1}$$

and basic questions like existence, uniqueness and regularity of solutions have been studied thoroughly in [1, 2]. While in general it is difficult to obtain quantitative properties of solutions to non-local nonlinear equations, Eq. (1) possesses special features that enable one to study the long term behaviours in terms of its self-similar solution. Using similarity variables motivated from the scaling relations, the transformed equation has an entropy function so that the convergence towards the self-similar profile in one dimension can be established by the well-known entropy method.
In this talk, we consider two variants of equation (1) with an absorption term or a convection term.

This talk is based on a joint work with Y. Huang and B. Volzone.

References

[1] L. A. Caffarelli and J. L. Vázquez. Asymptotic behaviour of a porous medium equation with fractional diffusion. *Discrete Contin. Dyn. Syst.*, 29(4):1393–1404, 2011.

[2] L. A. Caffarelli and J. L. Vázquez. Nonlinear porous medium flow with fractional potential pressure. *Arch. Ration. Mech. Anal.*, 202(2):537–565, 2011.