A Schwartz-type boundary value problem for monogenic functions in a biharmonic algebra

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We consider a commutative algebra $\mathbb{B}$ over the field of complex numbers with a basis $\{e_1, e_2\}$ satisfying the conditions $(e_1^2 + e_2^2)^2 = 0$, $e_1^2 + e_2^2 \neq 0$. Let D be a bounded simply connected domain in the Cartesian plane xOy and $D_\zeta = \{xe_1 + ye_2 : (x, y) \in D\}$. Components of every monogenic function

$$\Phi(xe_1 + ye_2) = U_1(x, y)\, e_1 + U_2(x, y)\, ie_1 + U_3(x, y)\, e_2 + U_4(x, y)\, ie_2$$

having the classic derivative in D_ζ are biharmonic functions in D, i.e., $\Delta^2 U_j(x, y) = 0$ for $j = 1, 2, 3, 4$.

We consider a Schwartz-type boundary value problem: to find a function $\Phi\colon D_\zeta \longrightarrow \mathbb{B}$ which is monogenic in a domain D_ζ when limiting values of components U_1, U_3 are given on the boundary ∂D_ζ:

$$U_1(x, y) = u_1(\zeta)\,, \quad U_3(x, y) = u_3(\zeta) \qquad \forall\, \zeta = xe_1 + ye_2 \in \partial D_\zeta.$$

This problem is associated with the following problem: to find a biharmonic function $V(x, y)$ in D when boundary values of its partial derivatives $\partial V/\partial x$, $\partial V/\partial y$ are given on the boundary ∂D. The problem is also associated with the *principal biharmonic problem*: to find a biharmonic function $V(x, y)$ in D, which is continuously extended together with partial derivatives of the first order up to the boundary ∂D, when its values and values of its outward normal derivative are given on ∂D.

Using a hypercomplex analog of the Cauchy type integral, we reduce the mentioned Schwartz-type boundary value problem to a system of integral equations and establish sufficient conditions under which this system has the Fredholm property. For a half-plane and for a disk, solutions are obtained in explicit forms by means of Schwartz-type integrals.

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