SOBOLEV EXTENSION OPERATORS IN CUSP DOMAINS

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Let Ω be a domain in the Euclidean space $\mathbb{R}^n$, $n \geq 2$. Consider extension operators on Sobolev spaces

$$E : L^1_p(\Omega) \to L^1_q(\mathbb{R}^n),\ 1 \leq q \leq p \leq \infty.$$

In the present work we prove the sharp necessary generalized Ahlfors type (p,q)-measure density condition for extension operators of seminormed Sobolev spaces:

Let there exists a continuous linear extension operator $E : L^1_p(\Omega) \to L^1_q(\mathbb{R}^n)$, $n < q \leq p < \infty$, then

$$\Phi(B(x,r))^{p-q}|B(x,r) \cap \Omega|^q \geq c_0|B(x,r)|^p,\ 0 < r < 1,$$

where Φ is a countable additive set function associated with the extension operator and a constant $c_0 = c_0(p,q,n)$ depends on p, q and n only.

In the case $q < p$ this condition allows Hölder cusp domains and in the case $p = q$ this condition coincides with the well known Ahlfors measure density condition.

References

[1] A. Ukhlov, Extension operators on Sobolev spaces with decreasing integrability, Trans. of A. Razmadze Math. Inst., (in press).